

Hopping conduction in NTD germanium: comparison between measurement and theory

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We present precise measurements of the resistance-temperature variation of several samples of neutron transmutation doped (NTD) germanium, at temperatures from 70 mK to 1 K. This material is widely used for sensitive thermometry, often as the thermistor element in bolometers and microcalorimeters. It is also used in investigations of the low temperature conductivity of highly doped semiconductors. The resistance, R , is expected to follow the variable range hopping equation $R(T) = R_0 \exp(T_0/T)^p$, where T is temperature and R_0 and T_0 are material parameters. A value of $p = 0.5$ is predicted theoretically, and generally seems to be in good agreement with experimental measurements. However, some theories and numerical calculations predict different values of p . Knowledge of the correct resistance-temperature relationship is important for the accurate calibration of thermometers, and also delivers insight into the basic physics involved. Most experimental measurements on germanium have not had sufficient precision to distinguish between the different predicted values of p . We show that such measurements are nevertheless possible. Our results are all in excellent agreement with the expected variable range hopping behaviour. However, the values of p appear to vary with doping density, in disagreement with most theories. We have considered and rejected both random and systematic errors as an explanation for the observed behaviour, and have confirmed the results by making measurements in two different systems with independent readout systems and temperature calibrations. The situation is complicated by the possibility of temperature dependence of R_0 . The expected form is $R_0(T) \propto T^q$; however, there is considerable disagreement over the predicted value of q . We show that in general it is not possible to determine both p and q from resistance measurements. However, our results can only be explained if either or both of q and p vary from sample to sample. Such behaviour is not generally expected. We show that neglecting the q term can lead to serious errors when calibrating thermometers. However, the degeneracy between p and q means that for a calibration the q term can be neglected, and good fits obtained if p is allowed to vary. Our results suggest that further theoretical work is required in this area, backed up by more comprehensive measurements.

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1. INTRODUCTION

Neutron transmutation doped (NTD) germanium semiconductors¹² are widely used as the thermistor element in bolometers³ and microcalorimeters,⁴ as well as for studying semiconductor properties. The neutron transmutation process results in semiconductors with excellent doping uniformity, while the doping concentration can be tuned precisely, allowing values extremely close to the metal-insulator transition to be achieved. This provides a well characterised system for investigating the low temperature conductivity of highly doped semiconductors. Doping close to the metal-insulator transition produces thermistors with low noise, high temperature sensitivity and excellent reproducibility.

At low temperatures, the resistance-temperature relationship of NTD Ge can be represented analytically to a high degree of accuracy. Determining the exact form of this relationship is of interest, since it gives information on the conduction mechanism, and can be used to derive information on the density of states.

In addition, it greatly simplifies the calibration of NTD Ge thermistors. Using an analytical calibration function has two main advantages. Firstly, it eliminates the need for complex empirical functions such as Chebychev polynomials to fit calibration data. This reduces the number of calibration points required since accurate interpolation is possible between widely spaced points. Extrapolation outside the calibrated temperature range is even possible with some confidence. Secondly, it simplifies the modelling of devices such as bolometers, and

enables analytical expressions to be written down for properties such as the responsivity.⁵

It is obviously important that the correct analytical expression is used. For the doping values generally employed for thermometry, the resistance-temperature relation is usually taken to be⁶

$$R(T) = R_0 \exp\left(\frac{T_0}{T}\right)^p \quad (1)$$

where R is the resistance at temperature T , and T_0 and R_0 are constants which depend on the doping and, for R_0 , on the thermistor dimensions. The exponent p is a constant, and it is often assumed that $p = 0.5$.

In this paper we present precise measurements on various types of NTD Ge which were taken as part of a program of bolometer and thermometer development. The results are in extremely good agreement with equation 1, but only if p is allowed to vary. We show that random and systematic errors are unlikely to have had a significant affect on our measurements of p . We consider the theoretical and experimental support for assuming that $p = 0.5$, and find it inconclusive, and discuss the implications of our measurements.

2. BACKGROUND

The conduction mechanism for NTD Ge thermistors at low temperatures is believed to be variable range hopping (VRH).

This involves thermally activated tunnelling of electrons between localised sites. Theories for VRH predict a general resistance equation⁷

$$R(T) = R_0 T^q \exp\left(\frac{T_0}{T}\right)^p, \quad (2)$$

where R_0 and T_0 are constant and q and p depend on the form of the electron density of states close to the Fermi energy. A consideration of long range interactions between the localised states suggests the existence of a gap in the density of states near the Fermi level, called the Coulomb gap (CG). Analytical calculations for this situation predict $p = 0.5$ (Ref. 8). Experimentally, results for NTD Ge are normally found which agree reasonably well with this value. It is therefore believed that the correct mechanism is CG VRH. It is then frequently concluded that a value of exactly $p = 0.5$ should be correct, with observed deviations from this value p attributed to experimental error. This conclusion is almost universal when the theoretical $R(T)$ relationship is used as a thermistor calibration function.

However, CG VRH theory has come under criticism.⁹⁻¹¹ Even assuming that the theory is correct and applicable in this case, there is considerable disagreement over the exact value of p . While the prediction of $p = 0.5$ is generally accepted, analytical calculations considering many electron transitions have predicted stronger, even exponential, temperature dependence.¹² Some numerical simulations have also suggested stronger temperature dependence, corresponding to $p = 0.55$ if interpreted as following a power law.^{13,14} Values of $p < 0.5$ are not usually found analytically or numerically.

Moreover, the value and even sign of q are uncertain,^{15,16} although the magnitude of the value is believed to be ≤ 1 . This factor is often ignored, or assumed to be negligible, although as will be shown later this assumption cannot generally be justified. We must therefore look to experimental data to determine the form of $R(T)$.

Unfortunately, power laws are hard to determine experimentally, and available measurements do not provide good evidence in favour of any of the suggested values of p . Results in the literature on NTD Ge are generally confined to plots of $\ln(R)$ against $1/\sqrt{T}$, showing that the results fall approximately on a straight line. However, plotting the data in this form can mask significant deviations from the fit.¹⁷ In a few cases, fits have been carried out to determine p . For NTD Ge, assuming $q = 0$, values of $p = 0.54$ (with a value of 0.5 fitting almost as well),¹⁸ and $0.5 - 0.55$ (Ref. 7) have been found. Direct measurements of the density of states found a temperature variation which would correspond to $p = 0.57$ (Ref. 19).

Various values have been measured for other systems believed to exhibit CG VRH. For doped semiconductors, these include $p = 0.46$ for CdSe (Ref. 20), while measurements presented on ion implanted Si:As (Ref. 21) can be fitted extremely well with $p \simeq 0.8$. Measurements on amorphous films have found $p = 0.56$ and $p = 0.57$ for indium oxide,²² and $p = 0.49$ (Ref. 22), $p = 0.5$ (Ref. 23) and $p = 0.72$ (Ref. 11) for NiSi.

It is rare for errors to be quoted for experimental determinations of p , and we are not aware of any papers in which the

effects of *systematic* errors are discussed. Accurate measurements of p would be of use in distinguishing between competing theoretical models. A prior knowledge of p would also be very useful for thermistor calibration. While it can be taken as a free parameter, knowledge of p reduces the number of data points required for an accurate calibration, and is particularly useful when extrapolation of the calibration data is required. While in principle one would carry out a calibration over the full temperature range required, using a large number of measurements, this is sometimes not practical.

Finally, it should be noted that there is little experimental evidence either for or against the existence of the q term. This is hard to detect experimentally, as will be shown later. Another complication is that at sufficiently high temperatures the hopping energy is expected to exceed the Coulomb gap, and Mott scattering with $p = 0.25$ is expected.^{24,25} Therefore for some temperature range, p is expected to cross over from $p = 0.5$ to $p = 0.25$.

3. MEASUREMENTS AND SAMPLES

The majority of the measurements reported in this paper were taken using a paramagnetic salt adiabatic demagnetisation refrigerator (ADR). Samples were mounted on a variable temperature stage which could be held at a constant temperature from 70 mK to around 1 K.¹ The stage was surrounded by a gold plated copper radiation shield at a temperature of approximately 1.6 K. The main magnet of the ADR is compensated to reduce the field at the position of the sample to a very low level – we calculate the value to be less than 10 mT at the highest fields used during measurements, and less than 3 mT for stage temperatures below 300 mK. All signal lines entering the cryostat include capacitive filters to reduce electromagnetic interference. To reduce the effect of microphonics, wiring in the dewar is formed of twisted pairs, held in place with GE 7031 varnish. A full description of the system is given in Ref. 26.

Two different readout methods were used to measure thermistor resistances. In both cases the measurements were made using a 4-wire configuration. For some measurements, the resistance was measured directly using an AVS-47 cryogenic resistance bridge.²

The remainder of the measurements were made by measuring the thermistor voltage as a function of current (a *load curve*), and then calculating the resistance. This was achieved by applying a variable bias voltage to the thermistor in series with two 60 M Ω load resistors. The voltage across the thermistor was then measured through a matched pair of IFN146 silicon JFET source followers, coupled to a low noise external amplifier. The load resistors were mounted on the stage to reduce Johnson noise. The voltage from the JFET amplifiers

¹ Note that this is a change from the system described in Ref. 26, for which the stage was not temperature controlled.

² RV-Elektronikka Oy, Vantaa, Finland.

Sample type	Ga doping density	Length	Cross-section
#12	$9.8 \times 10^{16} \text{ cm}^{-3}$	250 μm	$250 \times 250 \mu\text{m}^2$
#19	$8.1 \times 10^{16} \text{ cm}^{-3}$	300 μm^a	$25 \times 100 \mu\text{m}^2$
D	$4.6 \times 10^{16} \text{ cm}^{-3}$	250 μm	$250 \times 250 \mu\text{m}^2$
F	$3.7 \times 10^{16} \text{ cm}^{-3}$	250 μm	$250 \times 250 \mu\text{m}^2$

^aElectrode spacing 200 μm

TABLE I: Details of the different types of sample used. All samples of a given doping density had the same geometry. The As doping density is 28.5% of the Ga density.

and the applied bias voltage were measured using digital voltmeters. The JFET output voltage has an arbitrary DC offset. This was monitored by measuring the output at zero bias at the start of each load curve. The readout system is described in more detail in Ref. 26.

Absolute thermometry was provided by a germanium secondary standard thermometer based (above 650 mK) on the ITS-90 temperature scale.³

The sample area is well shielded from optical radiation. Extensive measurements in this system for bolometer characterisation have shown that the readout systems are stable, reproducible and well understood.²⁶

We also present measurements taken at a few stage temperatures in a dilution refrigerator system. Load curve measurements were made using a similar readout system to the ADR. However, the temperature calibration and readout systems in this system are entirely independent of the ADR.

The thermistors in all the samples were made using the NTD process from germanium with a natural isotope ratio.¹ This produces a p-type semiconductor (Ga acceptors with As compensating donors) with a compensation ratio of approximately 30%. Information on the different NTD germanium types used is given in Table I, and details of each sample are shown in Table II.

Two types of sample were measured. Some were packaged as thermometers; in this case, the germanium chip was epoxied to an electrically insulating substrate (quartz or sapphire) giving thermal contact to the stage. In all cases electrical contacts were made on opposite faces of each chip; the chip dimensions are given in Table I, where the length refers to the distance between the contacts. We have found that thermometers using this design maintain thermal equilibrium with their surroundings even at temperatures below 100 mK.

The remaining samples were used as the thermistor element in silicon nitride spiderweb bolometers.^{27,28} In this case the germanium was indium bump bonded to a silicon nitride membrane as described in Ref. 28. Thermal contact to the stage was via this membrane, forming an intentionally weak thermal link. Although not an ideal configuration for determining zero-bias resistance, this did not prevent us from making accurate measurements. The chips are approximately 300 μm long with a cross-section of $100 \times 25 \mu\text{m}$. Both con-

tacts, each 100 μm wide and 50 μm long, were made on the same side of the chip, separated by 200 μm .

4. RESULTS AND ANALYSIS

The resistance in equations 1 and 2 is the thermistor resistance as the bias tends towards zero. At finite bias the resistance can be affected by self-heating, in which the bias power dissipated in the thermistor raises the temperature above that of the substrate. A similar effect (electron-phonon decoupling)²⁹ may occur in which the electron temperature is raised above that of the thermistor lattice. Electric field effects, in which hopping is no longer purely thermal, can also occur for sufficiently high bias.²⁹

For each sample, the zero-bias resistance was calculated as a function of temperature. For measurements using the resistance bridge, this was just the measured resistance. We ensured that we were measuring the true zero-bias resistance by adjusting the excitation voltage setting on the bridge and checking that the measured value did not change.

For load curve measurements, the zero-bias resistance, R_0 , was taken from the low current (constant resistance) portion of each load curve. The constant resistance region was large enough to make an unambiguous determination of the zero bias resistance even for the bolometers. Measuring load curves gives a much better confirmation that self-heating effects have been avoided than with conventional resistance bridge measurements. We are thus confident that our values for R_0 are accurate even for the bolometers, which are strongly affected by self-heating.

To ensure that the above procedure provided accurate values for R_0 , fits were carried out to each load curve. For the bolometers this was done by modelling the properties of the weak thermal link to the stage,⁵ and for the thermistors a model for electric-field effects²⁹ was used. In both cases the models provided good fits to the data. The resulting values of R_0 were close to those derived as above, and using them would not significantly alter the derived values for p . Since the models used fitted the data well, we are confident that our results are not distorted by effects not included in the models such as electron-phonon decoupling, or electric-field effects in the bolometers.

Figure 1 shows the results for some of the samples. Equation 1 was fitted to the data for each sample, using a non-linear least squares fitting method with R_0 , T_0 and p as free parameters. The error in our resistance measurements is approximately 0.5%, over the full range of resistances. Points were therefore given equal weighting in the fits, which were carried out in terms of $\log(R)$. The results are shown in Table II. To ensure robustness of the fit, for each measurement in the ADR the fit was repeated several times, omitting points from both ends of the temperature range. In each case, similar values for p were obtained.

The errors quoted for p are taken from the least squares fits. As a check on these error values, for each measurement in the ADR the residuals were plotted for fits using various fixed values of p , and examined for signs of systematic deviation.

³ Lake Shore Cryotronics Inc, Westerville, Ohio, model GR-200A-30.

Name	NTD type	Package	Measurement	Temp. range	T'_0	T_0	p	q
12-1	#12	thermometer	AVS	100-1000 mK	10.24	5.65	0.5611 ± 0.0004	0.341 ± 0.002
12-2	#12	thermometer	AVS	100-1000 mK	14.48	10.79	0.525 ± 0.001	0.183 ± 0.004
12-3	#12	thermometer	AVS	100-1000 mK	13.91	8.20	0.548 ± 0.001	0.339 ± 0.005
19-1	#19	bolometer	VI	100-350 mK	13.18	13.20	0.500 ± 0.007	0.002 ± 0.001
19-2	#19	bolometer	VI	100-350 mK	12.86	16.43	0.481 ± 0.008	-0.147 ± 0.058
19-5	#19	bolometer	VI,D	100-1000 mK	16.69	23.56	0.474 ± 0.002	-0.200 ± 0.014
19-6	#19	bolometer	VI,D	100-1000 mK	16.26	23.65	0.472 ± 0.001	-0.215 ± 0.011
D-1	D	thermometer	VI	300-1000 mK	46.02	66.19	0.475 ± 0.005	-0.264 ± 0.046
D-2	D	thermometer	VI	300-1000 mK	45.73	65.02	0.475 ± 0.003	-0.257 ± 0.026
F-1	F	thermometer	VI	300-1000 mK	62.21	75.46	0.487 ± 0.002	-0.152 ± 0.022

TABLE II: Details of the samples. In the measurement column, VI (or AVS) indicates ADR load curve (or bridge) measurements, and D indicates dilution refrigerator measurements. The temperature range given is that over which fits were carried out. The T'_0 column gives the value of T_0 for fits using equation 1 with $p = 0.5$. The next two columns show T_0 and p for fits using equation 1 with p allowed to vary. The final column shows q for fits using equation 2 with $p = 0.5$ and q allowed to vary.

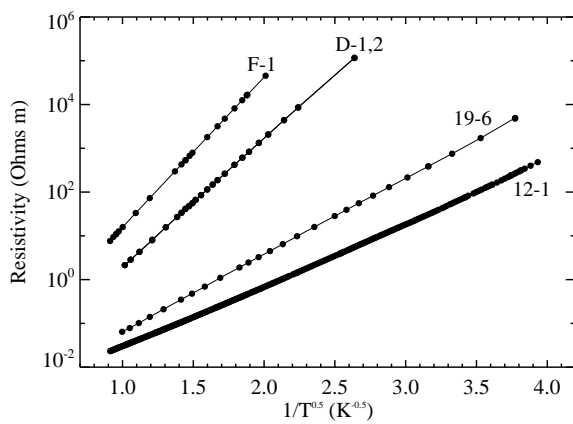


FIG. 1: Zero-bias resistance as a function of temperature for several different samples measured in the ADR.

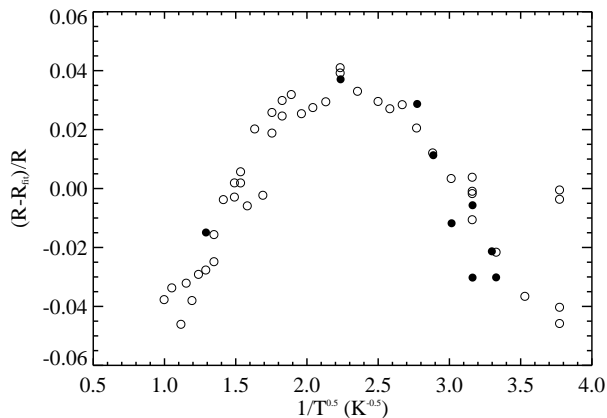


FIG. 2: Deviations from fits to data from sample 19-5, using equation 1 with a fixed value of $p = 0.5$. Measurements shown are from the ADR (\circ), and dilution refrigerator (\bullet). The best fit to this data is for $p = 0.47$.

In each case, the point at which such deviation was seen was in good agreement with the error values. Figure 2 shows how easily deviations from the standard fit with $p = 0.5$ can be seen by eye. Load curves from the dilution refrigerator were taken at too few temperatures to allow fitting by eye or varying the number of points taken for the fit.

Our measured points are not evenly spaced in temperature; fits to the whole temperature range did not depend significantly on whether every point, or a subset of points, equally spaced in temperature, were used.

As a further check, the results were plotted using the parameter w , where

$$w = -\frac{T}{R} \frac{dR}{dT}. \quad (3)$$

If equation 1 is correct, then

$$w = p \left(\frac{T_0}{T} \right)^p, \quad (4)$$

and

$$\log_{10}(w) = \log_{10}(pT_0^p) - p \log_{10}(T); \quad (5)$$

p can then be obtained as the gradient of a plot of $\log_{10}(w)$ as a function of $\log_{10}(T)$.

Since obtaining w involves numerical differentiation of experimental data, this technique is only possible for datapoints which are spaced sufficiently closely. For our results, this was true only for the measurements on NTD types #12 and #19. However, the resulting values for p were in good agreement with those obtained above, with no sign of p varying with temperature. Examples are shown in Fig. 3.

Fits were also carried out using equation 2 allowing q to vary and fixing $p = 0.5$. In addition, we fitted each sample assuming $p = 0.5$ and $q = 0$, using a linear least-squares fitting routine. We denote the value of T_0 resulting from these fits as T'_0 . Since the value of T_0 depends strongly on p , and in every case the linear fits produced reasonable agreement, we use T'_0 (shown in Table II) as a parameter to characterise the different samples.

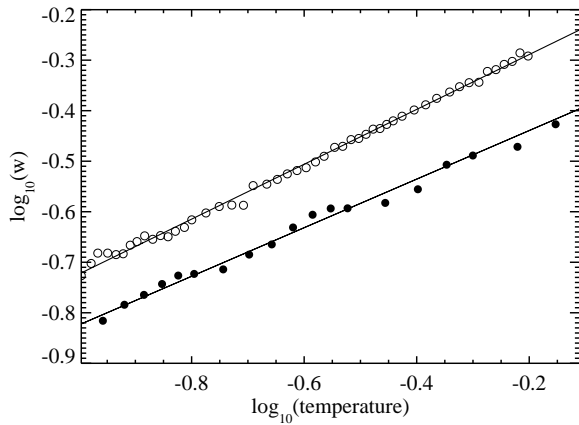


FIG. 3: Plots of the parameter w (see text) as a function of temperature, for samples 12-1 (\circ) and 19-6 (\bullet). The solid lines show linear fits to each dataset; the gradient gives the value of p .

We have considered various possible systematic sources of error in our measurements. The most likely potential source of error in the load curves is an incorrect value for the gain of the JFET amplifiers. The value of the gain has been measured to 1%. As a check, we have analyzed the data using different values for the gain. The effect of an increase or decrease in the gain is to change the value of p by an amount which decreases with increasing temperature. For example, for sample 19-6, changing the gain by 20% from the measured value causes negligible changes in p above 150 mK, and strongly diverging values below this temperature. It thus seems unlikely that our results are affected by a constant error in the gain. Similar effects were seen when the value used for the applied bias voltage was varied from the measured value. An error in the value used for the load resistors, although affecting the measured values for resistance, does not alter derived values for p .

Further evidence that the results are not an artefact of the readout system, such as a voltage dependent gain, comes from simultaneous resistance bridge measurements on an NTD sample and a commercial germanium standard thermometer. Despite the two thermometers having very similar resistances, and thus presumably being susceptible to similar systematic errors, they have different values for p . This can be determined by comparing the resistances without the calibration of either thermometer being required, demonstrating that calibration errors are not responsible for the varying values of p observed.

Our results are also supported by excellent agreement between measurements on the same sample in the ADR and dilution refrigerator over nearly four orders of magnitude in resistance. This shows that the readout electronics and temperature calibration are both in good agreement with our system. Good agreement was also seen when the same sample was measured more than once in the ADR.

We see no evidence of the magnetic field from the ADR magnet affecting the results. Such an effect should be seen

as a difference between measurements taken at the same stage temperature but different main magnet fields. (Cooling using an ADR is a single-shot process; as heat leaks into the stage the field required for a given temperature decreases). It should also be evident as a difference from the results taken in the dilution fridge, which does not use a magnet. In any case, we would not expect to see magnetic field effects since the calculated field at the sample (see section 3) is so small.¹⁸

Measurements at stage temperatures below 100 mK were not used for fitting. For many samples, it was difficult to obtain the zero-bias resistance accurately at such temperatures. In addition, for some measurements there appear to be errors in the thermometry at these temperatures.

5. DISCUSSION

The accuracy of fitting equation 1 to the data is shown in Fig. 4a. For every sample, the fits are extremely good. For similar measurements on ion-implanted silicon, good fits could only be produced over a smaller temperature range than shown here.³⁰ However, our fits required p to vary from the conventional value of $p = 0.5$. Figure 4b shows the results if it is assumed that $p = 0.5$. Allowing p to vary, the reduced chi squared values for the fits were mostly of order 1, suggesting that equation 1 (with p allowed to vary) is appropriate (see Fig. 5a).

The variation in values of p between the samples is much larger than the errors in p . We therefore conclude that the variation is real, and intrinsic to the samples. The observed values do not appear to be the result of a ‘cross-over’ between two different values of p as the temperature is altered, since p was not seen to vary with temperature (Fig. 3).

Different samples of the same NTD type appear to have similar values for p . This can be seen from Fig. 5b, showing the fitted values for p as a function of T'_0 , which is inversely proportional to doping density. This suggests that the values of p may be intrinsic not just to each sample, but each NTD type. However, there is some variation in the values of p for a given type which appears to be larger than can be accounted for by random errors.

Theories predict values of $p \gtrsim 0.5$. Thus the observation of $p \simeq 0.47$ for some samples is somewhat surprising. (It should be noted that while this value is very close to $p = 0.5$, the errors are much smaller than the difference from 0.5, and indeed the deviation from a fit with $p = 0.5$ is easily visible by eye (see Fig. 2).) Moreover, while different theories predict different values of p , a value which varies from sample to sample is unexpected. (It has, however, been suggested that the value of p may depend on the degree of disorder.^{13,14}) However, the analysis is complicated by the possible existence of a temperature-varying prefactor ($q \neq 0$). Unfortunately, the measurements do not allow us to determine if such a term exists. From Fig. 5a it can be seen that there is no reason to favour fits in which p varies over those with q varying.

This is a general problem; for a wide range of values of T'_0 , p and q , measurements can be fitted to a high degree of accuracy allowing only p to vary. However, the resulting value

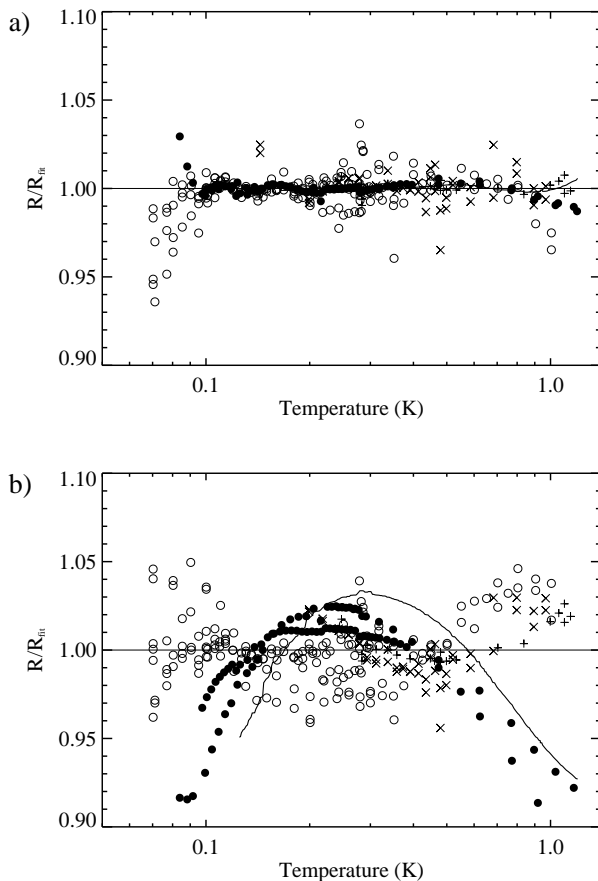


FIG. 4: Accuracy of fits to equation 1 for the different samples with a) p allowed to vary and b) $p = 0.5$. The types are #12 (\bullet and line), #19 (\circ), D (\times) and F ($+$).

of p can be very different from that obtained if the true value of q was used. This is illustrated in Fig. 6. Here, simulated datasets for $p = 0.5$ and various values of q are fitted assuming $q = 0$, but allowing p to vary. The figure shows the values of p that are obtained, for fits over two different temperature ranges. Note that the results do not depend strongly on the temperature range chosen. The results are independent of the value of R_0 .

Similar results are obtained for fixed values of p other than 0.5. For our measurements, fits allowing both p and q to vary produced similar chi squared values to those shown in Fig. 5a.

Values of q predicted on theoretical grounds include $q = 1$ (or, more generally, $q = 2p$) (Ref. 31) and $q = 0.5$ (Ref. 16).

We are aware of very few attempts to measure q . Measurements on uncompensated NTD Ge near the metal insulator transition (MIT)⁷ show constant values of $p \simeq 0.5$ approaching the MIT only for $q = -1/3$. This value was suggested by comparison with behaviour on the metal side of the MIT. Later work on compensated NTD germanium³² also supports this value for q .

In Ref. 33 it is claimed that $q = 2p$ gave the best fit to data on ion implanted Si:As, although with “substantial” error

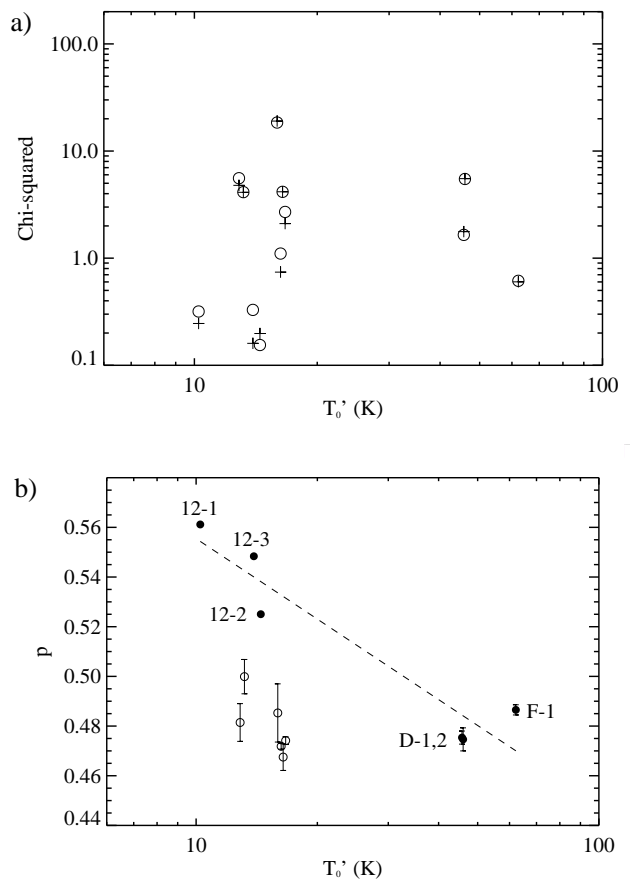


FIG. 5: a) Values of chi-squared for fits using equation 1 allowing p to vary (\circ) and to equation 2 allowing q to vary and fixing $p = 0.5$ ($+$). b) Measured values of p as a function of T_0' for bolometers (\circ ; type #19) and thermistors (\bullet ; sample names shown). The dashed line shows a linear least-squares fit to the thermistor points.

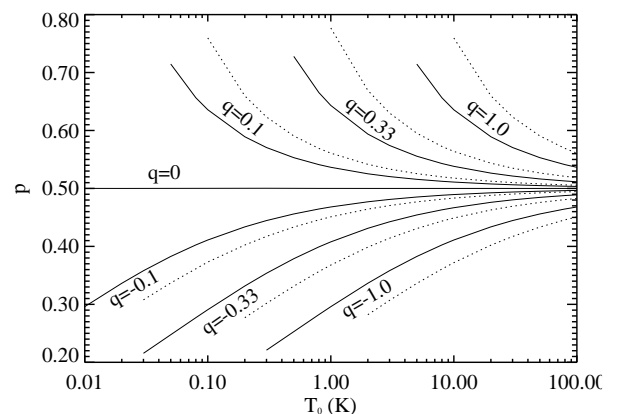


FIG. 6: Results of fitting datasets simulated using equation 2 for $p = 0.5$ and various values of q , then fitting them assuming $q = 0$ and allowing p to vary. Fits were carried out over the range 20 to 100 mK (solid lines) and 100 mK to 1 K (dashed lines).

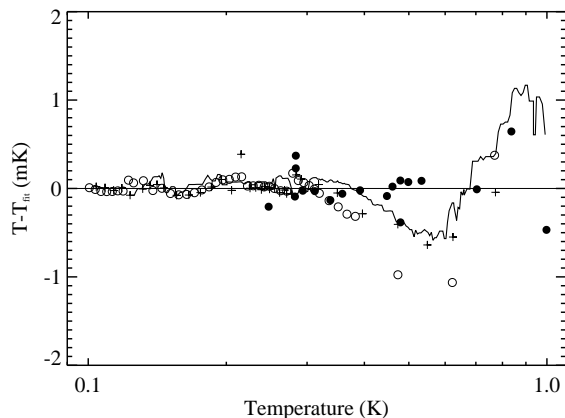


FIG. 7: Accuracy of fits to equation 1 with p allowed to vary, in terms of temperature. The samples are 12-1 (line), 12-2 (\circ), 12-3 ($+$) and F-1 (\bullet).

bars. However, in measurements³⁰ on ion implanted Si:P and Si:B, a value of $q = 2p$ was found to *decrease* the quality of fits. Measurements on CuInTe₂ (Ref. 34) could only be fitted by using a non zero value of q ; a value of $p = 0.5$ could then be obtained with a value of $q = 0.18$.

It can be seen from Fig. 6 that for the range of T'_0 values for our samples, the values of q suggested above have a significant effect on the values of p obtained by assuming $q = 0$. The effects of a term in q can therefore not be ignored.

A negative value of q could explain our observation of samples in which $p < 0.5$. However, applying this (or any other) value for q for each sample still causes p to vary between samples. The same is true if we assume $q = 2p$ as in Ref. 33. A failure to obtain the true zero-bias resistance would also lower the measured value of p ; however, such effects do not appear to be significant for our measurements (see section 4).

We must therefore assume that at least one out of p and q is varying from sample to sample, and are unable to determine which from the data. This suggests that the usual theories, which predict fixed (albeit not well determined) values of p and q are too simplistic. While variation of p and q with parameters such as the degree of disorder¹⁴ and the doping density¹⁵ have occasionally been suggested, we are not aware of any quantitative predictions.

In our results, there is a suggestion of a trend for p (or, equivalently, q) to decrease with increasing T'_0 (see Fig. 5b). However, if this is the case, the results for NTD type #19 are anomalous. These samples are packaged as bolometers, and consequently mounted in a different fashion from the remainder of the samples. Mechanical stress caused by differential thermal contraction is believed to be capable of changing the properties of NTD Ge samples.³⁵ However, we do not think this is likely to be the cause of our results. Care is taken when mounting the thermometers to avoid the occurrence of stresses, by using a small epoxy contact area. Further, there is very good agreement between values of T'_0 (and p) for different samples of the same type; it is unlikely that the effects

of stress would occur with such uniformity. Another difference between these samples and the remainder is the geometry (they have a much larger length compared with their cross-sectional area) and location of the contacts on one face, rather than on opposite faces (see section 3).

Further systematic measurements on a range of NTD types would help determine if there really is such a trend, and could provide useful input for further development of the theories.

Our results have important consequences for calibration of NTD thermometers. They clearly suggest that p should be taken as a fitting parameter. Figure 7 shows the accuracy that can be achieved using equation 1 in terms of temperature. It can be seen that over most of the temperature range the accuracy is much better than ± 1 mK. While the value of q is unknown, in practise it is usually possible to ignore this term, with any effect taken up by a variation in p . If p does indeed depend on the doping density, or even is constant for a particular wafer, it may be possible to carry out fits using a known value of p . This would be useful since a relatively large number of points are required to constrain p well. If p is known, good fits can be carried out with a small number of points.

Finally, we note that a change in the value of T'_0 for the same sample measured on separate occasions (in different refrigerators) has been reported.³⁶ A sensitivity to thermal history was proposed as a possible explanation. We see no evidence of such behaviour - all measurements on particular samples on separate cooldowns, and in different facilities, are in extremely good agreement.

6. SUMMARY

We have shown that it is possible to make sufficiently precise resistance measurements on NTD germanium to distinguish between the values of p (in equation 1) predicted by different theories. Equation 1 was a very good fit to all the data; however, the value of p varied between samples. We believe the measured values of p are intrinsic to the samples, and not the result of random or systematic errors.

Our results are therefore in excellent agreement with the predictions of variable range hopping theory. However, they do not appear to agree with the classical Coulomb gap VRH prediction of $p = 0.5$. Neither do they agree with other Coulomb gap theories, since although such theories predict different values of p , the value is not expected to vary from sample to sample. Moreover, for some samples we obtain values of $p < 0.5$, which is not expected theoretically.

However, a pre-exponential term (q , equation 2) is predicted with an unknown sign and value. We have shown that it is very hard to distinguish experimentally between the effects of the p and q term since they are nearly degenerate. Nevertheless, our results can only be accounted for if either p , q , or both can vary from sample to sample. It would therefore seem that none of the theories currently available can fully account for the observed behaviour.

More systematic measurements on samples with a range of doping densities (and perhaps compensation levels) could provide useful input to the theories. Such measurements should

also show if there is a relationship between p and doping density as suggested by our results.

In terms of using NTD Ge as a thermistor material, we have shown that the common assumption that $p = 0.5$ and $q = 0$ leads to unnecessarily poor fits. However, extremely good calibrations can be obtained using equation 1 and allowing p to vary, since this allows for the possibility of a non-zero value of q .

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