Predicting the thermal conductivity of aluminium alloys in the cryogenic to room temperature range

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Aluminium alloys are being used increasingly in cryogenic systems. However, cryogenic thermal conductivity measurements have been made on only a few of the many types in general use. This paper describes a method of predicting the thermal conductivity of any aluminium alloy between the superconducting transition temperature (approximately 1 K) and room temperature, based on a measurement of the thermal conductivity or electrical resistivity at a single temperature. Where predictions are based on low temperature measurements (approximately 4 K and below), the accuracy is generally better than 10%. Useful predictions can also be made from room temperature measurements for most alloys, but with reduced accuracy. This method permits aluminium alloys to be used in situations where the thermal conductivity is important without having to make (or find) direct measurements over the entire temperature range of interest. There is therefore greater scope to choose alloys based on mechanical properties and availability, rather than on whether cryogenic thermal conductivity measurements have been made. Recommended thermal conductivity values are presented for aluminium 6082 (based on a new measurement), and for 1000 series, and types 2014, 2024, 2219, 3003, 5052, 5083, 5086, 5154, 6061, 6063, 6082, 7039 and 7075 (based on low temperature measurements in the literature).

Keywords: metals (A); thermal conductivity (C); electrical conductivity (C)

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I. INTRODUCTION

Aluminium alloys are increasingly used in cryogenic systems and particularly - due to their low density - in aerospace applications. Despite this, information on their thermal conductivity at low temperatures is limited. There are many different types of alloy in general use, and cryogenic thermal conductivity measurements have been made on only a few. The situation is complicated by the fact that readily available aluminium alloys vary from country to country. Since the majority of measurements have been made in the USA, this is of particular concern to workers in other parts of the world.

Making accurate thermal conductivity measurements over the temperature range from 1 K to room temperature is difficult and time consuming. However, it is well known that for most metals at liquid helium temperatures the Wiedemann-Franz law [1] can be used to convert electrical resistivity measurements into thermal conductivity. This is extremely useful, since it is much easier to measure resistivity than thermal conductivity. At higher temperatures, though, converting between the two quantities is not straightforward.

Hust et al. [2] have presented a set of equations for *pure* aluminium which enable predictions of the thermal conductivity to be made based on a measurement of the thermal conductivity or electrical resistivity at a single temperature. The resulting values are valid for temperatures from the superconducting transition temperature (T_c) to room temperature and above. Here, I show that a modified version of these equations enables similar predictions to be carried out for aluminium alloys (defined here as being of "commercial" purity (1000 series) or less). While the greatest accuracy is achieved by using low temperature measurements, useful values can also be produced from room temperature measurements for most aluminium alloys.

This enables aluminium alloys to be used with confidence in applications where the thermal conductivity is important, even when no direct measurements are available. Where measurements have been made, these equations can provide a good check on the likely accuracy of the measurement.

Readers wishing to apply the methods described in this paper and willing to trust the reasoning may continue reading at section V, which is reasonably self-contained. In addition, a conductivity calculator using the results from this paper is available on the world wide web at http://links.lowtemp.org/alkappa.html.

II. EQUATIONS

Hust et al. [2] present a set of semi-empirical equations which can be used to describe the normal state thermal conductivity of pure metals from arbitrarily low temperatures to room temperature and above. The thermal conductivity is given by

$$\kappa = \frac{1}{W_0 + W_i + W_{i0}}, \tag{1}$$

where W_0 and W_i represent the thermal resistance due to electron-defect and electron-phonon interactions respectively. Hence W_0 depends on the chemical purity and physical structure of the material, while W_i is, to a good approximation, an intrinsic property of a given metal. The final term, W_{i0}, is necessary to produce acceptable fits. These terms are given by:

$$W_0 = \frac{\beta}{T}, \tag{2}$$

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$$W_i = P_1 T^{P_2} \times \left[1 + P_1 P_3 T^{(P_2+P_4)} \exp\left(-\left(\left(\frac{P_5}{T}\right)^{P_6}\right)\right)\right]^{-1} + W_c \quad (3)$$

and

$$W_{i0} = P_7 \frac{W_i W_0}{W_i + W_0}, \quad (4)$$

where T is temperature. The parameter β varies with the purity of the metal, and can be written as a function of the residual resistivity ρ_0 (the constant value of resistivity in the limit of zero temperature):

$$\beta = \frac{\rho_0}{L_0}, \quad (5)$$

by applying the Wiedemann-Franz law [1]

$$\kappa = \frac{L_0 T}{\rho}, \quad (6)$$

where ρ is resistivity and L_0 is the Lorenz number (2.45×10^{-8} WΩK^{-2}).

Metal conductivity is often characterised by the residual resistance ratio (RRR); this is the ratio of room temperature to residual resistance, and is thus related to β by

$$\mathrm{RRR} = \left(\frac{\rho_{RT}}{L_0}\right)\frac{1}{\beta}, \quad (7)$$

where ρ_{RT} is the room temperature resistivity. Since the variation of ρ_{RT} with purity is very small for pure aluminium, β can be taken to be inversely proportional to RRR to a very good approximation. This is not true for alloys, and for convenience results are expressed here in terms of equivalent residual resistivity RRR^*, defined by

$$\mathrm{RRR}^* = \frac{\rho_{RT\_pure}}{\rho_0}, \quad (8)$$

where ρ_{RT_pure} is a value for the room temperature resistivity of *pure* aluminium, taken here as $2.43 \times 10^{-8}\,\Omega$m [3]. RRR^* is thus inversely proportional to β.

Table I: Fit parameters [2] for the thermal conductivity of pure aluminium using equations 3 and 4.

Parameter	Value
P_1	4.716×10^{-8}
P_2	2.446
P_3	623.6
P_4	-0.16
P_5	130.9
P_6	2.5
P_7	0.8168

Note that the choice of the value for ρ_{RT_pure} does not affect the results quoted here; it just alters the scaling of the quoted values for RRR^*.

Returning to Eq. 1, the parameters P_1 to P_7 can be determined for a given metal by least squares fitting to measurements. The W_c term is a function of temperature chosen empirically to improve the accuracy of the fits.

The parameters determined for pure aluminium [2] are given in Table I. Fits using these parameters were then compared with experimental data for high purity samples to obtain an expression for W_c:

$$W_c = -0.0005 \ln\left(\frac{T}{330}\right) \exp\left[-\left(\frac{\ln\left(\frac{T}{380}\right)}{0.6}\right)^2\right] - 0.0013 \ln\left(\frac{T}{110}\right) \exp\left[-\left(\frac{\ln\left(\frac{T}{94}\right)}{0.5}\right)^2\right] \quad (9)$$

These results were obtained by carrying out fits to data from the literature for relatively large, well annealed specimens. However, it was found that these values also produced good fits to data for *unannealed* specimens, being accurate to $\pm 20\%$. The fits are applicable to samples with residual resistivity values from RRR=10 to RRR=10 000.

These equations do not apply to aluminium in the superconducting state. The superconducting transition temperature (T_c) of aluminium alloys depends strongly on the extent to which each impurity is in solid solution [4, 5]. Predicting T_c for a commercial alloy is difficult since there has been little work on the effect of the presence of many different impurities, as is found in commercial alloys. In addition, the amount of each impurity in solid solution is usually not well known. The small number of reported measurements [6–13] on commercial alloys give values of T_c varying from below 400 mK to 1.4 K. No particular pattern is evident other than that the highest values appear to correspond to aluminium-silicon and aluminium-lithium alloys; the highest value for other alloy types is approximately 1 K.

III. FITTING

A literature search was carried out to obtain a database of thermal conductivity measurements for commercially available aluminium alloys. Several compilations [16, 17, 20, 21] were consulted in addition to original papers. The database was restricted to sets of data which covered a temperature range extending below 30 K and above 100 K, and thus included the conductivity peak. In total, 29 datasets were found which met this criterion; these are referred to below as database 1. Where possible, the temper (cold working and/or heat treatment) of the samples was noted, since conductivity can vary significantly with temper. These datasets covered a range of RRR^* from 0.8 to 13.8, overlapping with the range in Ref. [2] ($\mathrm{RRR} \simeq 10$ and above).

Different attempts at using Eq. 1 to fit the measurements are shown in Table II. In general, attempts at fits were either

Table II: The different fits carried out

Row	Fixed parameters (and values)	Global fit parameters[a]	Individual fit parameters[b]	W_c[c]	Database	Success[d]
1	P_1 to P_7 (Table I)	—	β	Yes	1	Poor
2	—	—	β, P_1 to P_7	Yes	1	24/29
3	—	—	β, P_1 to P_7	Yes	2	15/15[e]
4	—	P_1 to P_7	β	Yes	2	Poor
5	—	P_1 to P_7	β	No	2	Poor
6	P_2, P_4 to P_7 (Table I)	—	β, P_1, P_3	Yes	2	12/15
7	—	P_2, P_4 to P_7	β, P_1, P_3	Yes	2	12/15[f]
8	—	P_2, P_4 to P_7	β, P_1, P_3	No	2	12/15[f]
9	P_2, P_4 to P_7 (Table I) P_1,P_3 (Eq. 10 & 11, Table IV)	—	β	Yes	2	12/15
10	P_2, P_4 to P_7 (Table I) P_1, P_3 (Eq. 10 & 11, Table IV)	—	β	No	2	12/15[g]

[a]These parameters were allowed to vary, but with the same value used for each dataset
[b]These parameters were fitted separately for each dataset
[c]Denotes whether the W_c term (equation 9) was included in equation 3
[d]A successful fit is defined as one where no point deviated from the fit by more than 10%. The same 12 datasets were successful for rows 6 to 10
[e]Database 2 consists of database 1 with all data from references [22] and [23] removed
[f]No significant change from row 6
[g]Quality of fits slightly better than for row 9.

successful or complete failures, and it was thus obvious which methods of fitting were appropriate.

Fits using parameters for pure aluminium were not successful (Table II, row 1). However, allowing all parameters to vary separately for each dataset produced good fits to most of the measurements (row 2). The five *unsuccessful* fits all corresponded to data obtained by one particular group [22, 23]. A full description of their experimental configuration was not given, and it seems plausible that the measurements suffer from significant experimental error. Therefore all measurements from this group were rejected from further consideration leaving 15 datasets remaining (database 2); these are listed in Table III along with further datasets not used at this stage of the analysis.

It was not possible to find any values of the parameters P_1 to P_7 which allowed good fits to be obtained by varying only β with each dataset (rows 4 & 5). However, good fits did result for most datasets if only β, P_1 and P_3 were allowed to vary for each dataset (rows 6-8). The remaining parameters were fixed, with a choice of the pure aluminium parameters being as good as allowing them to take other values. It is therefore possible to make a three parameter fit to each dataset. Furthermore, plotting P_1 and P_3 as a function of RRR^* (equivalent to β) shows a systematic dependence. This is shown in Figure 1, along with fits to the data.

Since equation 1 is not very sensitive to the values of P_1 and P_3, the exact form of the fits is not important. Power-laws were chosen since they were the simplest functions which produced acceptable fits. The equations used for P_1 and P_3 are

$$P_1(\mathrm{RRR}^*) = \min\left(\alpha_1 \left(\mathrm{RRR}^*\right)^{\beta_1}, P_{1\_pure}\right) \qquad (10)$$

and

$$P_3(\mathrm{RRR}^*) = \max\left(\alpha_3 \left(\mathrm{RRR}^*\right)^{\beta_3}, P_{3\_pure}\right) \qquad (11)$$

where P_{1_pure} and P_{3_pure} are the pure aluminium values for P_1 and P_3 from Table I. These values are used for sufficiently high RRR^* so that the equations can be used for pure aluminium as well as alloys. Values for the remaining coefficients are given in Table IV; these were obtained by carrying out fits as in row 6 of Table II, except that P_1 and P_3 were obtained using equations 10 and 11, with the parameters α_1, β_1, α_3 and β_3 chosen to produce the best fits.

Single parameter fits can thus be carried out for aluminium alloys in a similar manner to pure aluminium; the only difference is that the parameters P_1 and P_3 must be taken as functions of β (Table II, rows 9 & 10). This extends the lower limit of fits from Ref. [2] from approximately $\mathrm{RRR} = 10$ to $\mathrm{RRR}^* < 1$, with a smooth transition into the new range of values.

Physically, the P_1 and P_3 terms describe the electron-phonon interaction (Eq. 3) [24]. While P_1 and P_3 are both required to vary, the P_3 term only appears in the equations as the product $P_1 P_3$, and the variation of this product is much smaller than that of P_1 or P_3 alone. Therefore the correction required to allow the equations for pure aluminium to represent aluminium alloys consists largely of simply multiplying the strength of the electron-phonon interaction by a purity

Table III: Datasets for the thermal conductivity of aluminium alloys used in this paper.

Number	Source	Alloy	Used?[a]	RRR^{*b}	1-K conductivity ($Wm^{-1}K^{-1}$)[c]
1	[14]	7039 T61	yes	1.42	1.43
2	[15]	6063 T5	yes	8.70	8.77
3	[15]	3003 F	yes	2.65	2.67
4[d]	[15]	2024 T4, 5083 O, 5086 F	yes	0.78	0.79
5	[15]	5052 O	yes	1.12	1.13
6	[15]	5154 O	yes	1.02	1.03
7	[15]	1100 F	yes	13.74	13.85
8	[15]	1100 O	yes	11.38	11.47
9	[15]	single crystal	no	88.86	89.95
10	[16]	4 S	yes	1.32	1.33
11	[16]	75 S	yes	—-	—-
12	[17]	2014 T6	yes	—-	—-
13	[17]	J-51	yes	8.68	8.75
14	[17]	5456 H343	yes	—-	—-
15	[17]	3004	yes	1.31	1.32
16	[17]	24 S	yes	0.88	0.89
17	[18]	5052 swaged	no	1.24	1.25
18	[18]	5052 annealed	no	1.3	1.31
19	[18]	2024	no	0.75	0.76
20	[17]	2219 T81	no	0.95	0.96
21	[17]	Duralumin as stamped	no	1.52	1.53
22	[17]	Duralumin	no	1.51	1.52
23	[19]	1050	no	18.9	19.1
24	[19]	6061 T6	no	2.29	2.31
25	[13]	6061 T6	no	2.5	2.52

[a]Denotes whether this dataset was used to derive fitting parameters (database 2 in Section III). Dataset 9 was not used since it represents pure aluminium (it is used to ensure that the results of the fitting could be applied successfully to pure aluminium as well as alloys). Datasets 17 to 25 did not cover a sufficiently large temperature range.

[b]This column gives the best fit values of RRR^* (defined in equation 8); rows with no entry correspond to datasets that could not be fitted.

[c]This is the conductivity at 1 K predicted by fits to the data using the RRR^* value given here, and not the measured value.

[d]The measurements on these three alloys are almost identical, and they are therefore represented by a single dataset

Table IV: Values of the coefficients in equations 10 and 11.

Coefficient	Value
α_1	2.958×10^{-8}
β_1	0.129
α_3	925.4
β_3	-0.167

dependent factor. Note that the temperature variation of the electron-phonon interaction is not altered at all from the pure aluminium values.

These fits are illustrated in Fig. 2, where fits (Table II, row 9) to all measurements from database 2 are shown. In each case, the fit is based on a *single* datapoint, with no attempt to allow for noise. In all but two cases the lowest temperature point was chosen. However, for two of the datasets in Fig. 2c, where the fits do not agree well with the data, a higher temperature point was chosen to improve the agreement between fit and measured data.

Despite basing the fits on a single point, the fits agree with the data to 11% or better (apart from the three datasets which did not fit well) - this is only slightly worse than the agreement when the parameters P_1 and P_3 were chosen individually for

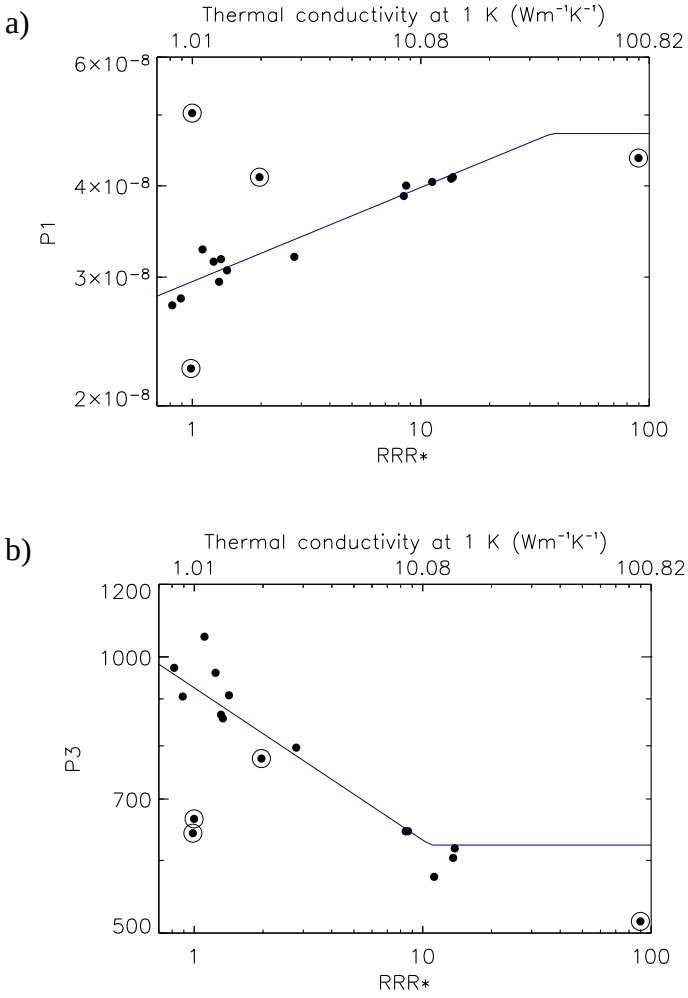

Figure 1: The fitting parameters P_1 and P_3 determined from fits to experimental data for aluminium alloys, as described in Section III. The solid lines show fits to the data, as described in the text. The circled points were excluded from these fits either because they corresponded to datasets for which the conductivity could not be fitted well or because the value of RRR^* was too high. Note that coincidentally for aluminium the conductivity at 1 K in SI units has almost exactly the same numerical value as the RRR^* value.

each dataset and fitted using every point. It seems likely that the three badly fitting datasets are due to experimental error. However, the possibility that some alloys deviate from these fits cannot be ruled out.

Fits were also carried out in the same manner to datasets rejected earlier because they did not cover a sufficiently large temperature range. The fits to all eight of these datasets (17-24 in Table III) also agreed with the data to 10% or better.

Repeating the fits shown in Fig 2 with the W_c term omitted (Table II, row 10) reduced the quality of fits, although fits to whole datasets (as opposed to a single point) were generally improved slightly.

Figure 3 shows fits to the datasets with $\mathrm{RRR}^* \leq 4$ which include room temperature values. Here, the fits are based en-

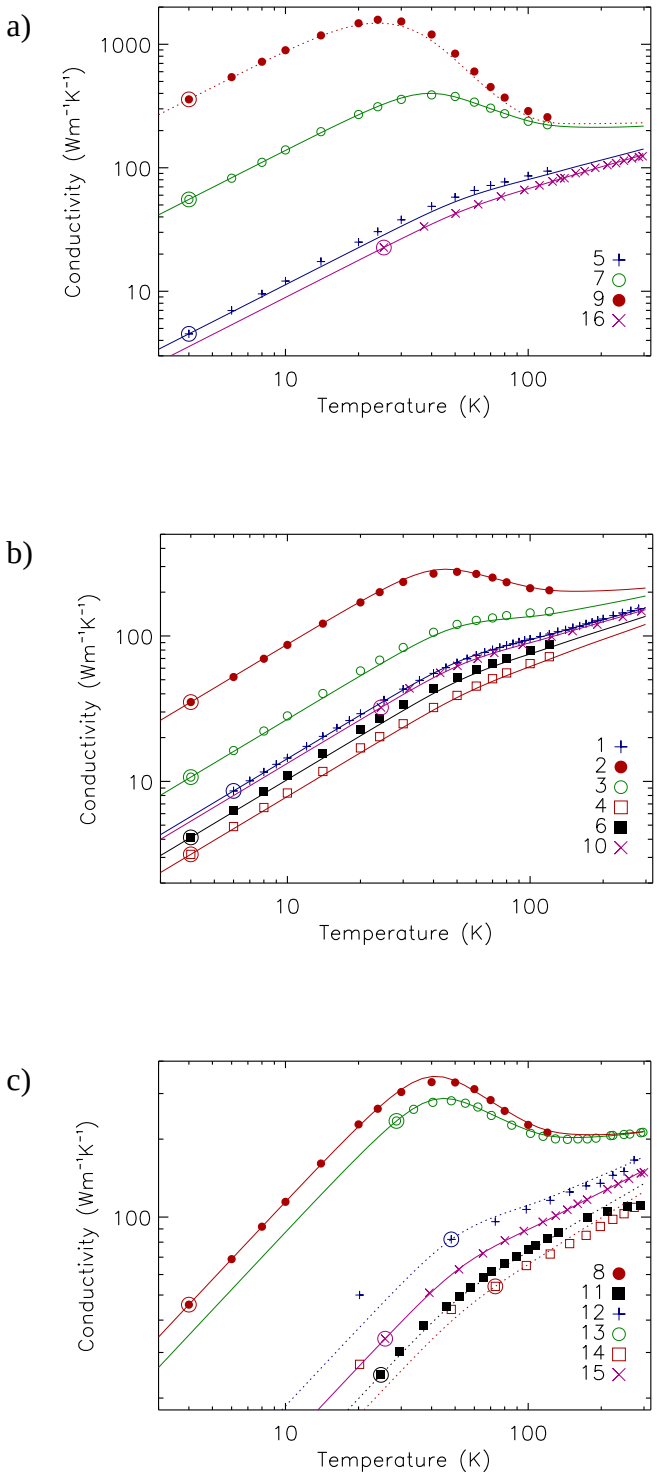

Figure 2: Single parameter fits to thermal conductivity measurements on aluminium alloys, as described in section III. Note that there is only one fitting parameter (RRR^*), and that each fit is based entirely on a *single* point in the corresponding data; this point is shown with a ring around it in the graphs.

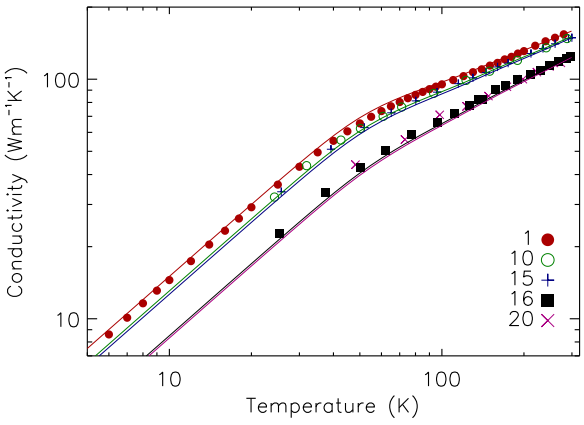

Figure 3: Single parameter fits to thermal conductivity measurements as in Fig. 2, except that each fit is based on the highest temperature point in the corresponding dataset rather than a low temperature point.

tirely on the highest temperature point. The worst agreement between data and fit is 12.5%, with the other four datasets agreeing to better than 10% for every point.

It should be noted that for the lower end of the range of RRR^* values, the lattice conductivity of aluminium alloys can become significant. While it is not easy to separate from the electronic conductivity, it appears that it can approach 15% or more of the total conductivity [15, 25]. Equation 1 does not explicitly include a lattice term. Including such a term would be difficult due to the lack of theoretical or experimental data on lattice conductivity [25]. Fortunately, the fits produced are sufficiently good that attempts to add a lattice term are unnecessary.

IV. USING ELECTRICAL RESISTIVITY MEASUREMENTS

In the previous section it was shown that thermal conductivity measurements can be extrapolated accurately to other temperatures. However, as described earlier, it is also possible to predict the thermal conductivity from electrical resistivity measurements, via the Wiedemann-Franz law (equation 6). At low temperatures (approximately 4 K and below), equation 6 is found to work well for most metals, with the Lorenz number L_0 taking the theoretically predicted value of 2.45×10^{-8} $\mathrm{W\Omega K^{-2}}$ [1].

Evidence that this is a good approximation for aluminium alloys is given in Refs [14], [15], [26] and [27]. Deviations of L_0 from the theoretical value were 10% or less, apart from two samples in Ref. [15] with deviations of about 12%. (Values for commercially pure alloys in Ref [27] show greater deviation - these can be neglected since the thermal and electrical measurements were made on different specimens from different sources, and the conductivity of these alloys varies significantly with exact purity and thermal history.) It is therefore straightforward to convert low temperature (residual) resistivity measurements to a thermal conductivity value which can then be used to predict the conductivity up to room temperature with reasonable accuracy. Some measurements from the literature are shown in Table V.

Since accurate thermal conductivity measurements are not straightforward to perform, and few results reported in the literature have been confirmed by measuring a standard reference material with the same equipment, the risk involved in relying on this method is probably no worse than using any but the most rigorous thermal conductivity measurements found in the literature.

As the temperature is increased above 4 K, the Lorenz number generally deviates from the theoretical value, returning to an approximation to this value near room temperature. For aluminium alloys, various contradictory expressions have been given for the room temperature Lorenz number [28]. Since the dependence of resistivity on purity is much smaller at room temperature than at low temperatures, this uncertainty prevents a direct conversion from resistivity to thermal conductivity from being accurate enough to usefully predict the low temperature conductivity from the measured room temperature resistivity.

However, there is an alternative approach. The room temperature resistivity ρ_{RT} can be approximated by Mathiessen's Rule:

$$\rho_{RT} \simeq \rho_i + \rho_0, \tag{12}$$

where ρ_i is the intrinsic resistivity, due to electron-phonon interactions, and ρ_0 is the residual resistivity, due to electron-defect interactions. Therefore so long as deviations from equation 12 and the variation of ρ_i with alloy purity are both sufficiently small, we can obtain ρ_0 by subtracting a value for ρ_i from the measured room temperature resistivity ρ.

Measurements on various aluminium alloys with a variety of tempers [14, 27] suggest that the difference between room temperature and residual resistivity can indeed be taken as a constant; this is shown in Figure 4. The variation in resistivity difference between the different alloys is small, with no evidence of systematic variation with resistivity. Taking the mean value for the resistivity difference, we can obtain residual resistivity from the room temperature value as

$$\begin{aligned} \rho_0 &\simeq \rho_{RT} - \rho_i \\ &\simeq \rho_{RT} - 2.64 \times 10^{-8} \Omega\mathrm{m}. \end{aligned} \tag{13}$$

This can then be converted to the low temperature thermal conductivity using the Wiedemann-Franz law (equation 6), and the thermal conductivity extrapolated to higher temperatures. For these measurements, "room temperature" was 273 K. The resistivity of pure aluminium increases by approximately $0.011 \times 10^{-8} \Omega\mathrm{mK}^{-1}$ with increasing temperature; the alloy measurements in Ref. [14] are in good agreement. Measurements of ρ_{RT} should be corrected to the 273 K value before using equation 13.

It should be noted that the value for ρ_i is somewhat larger than the resistivity of pure aluminium (for which the ρ_0 term

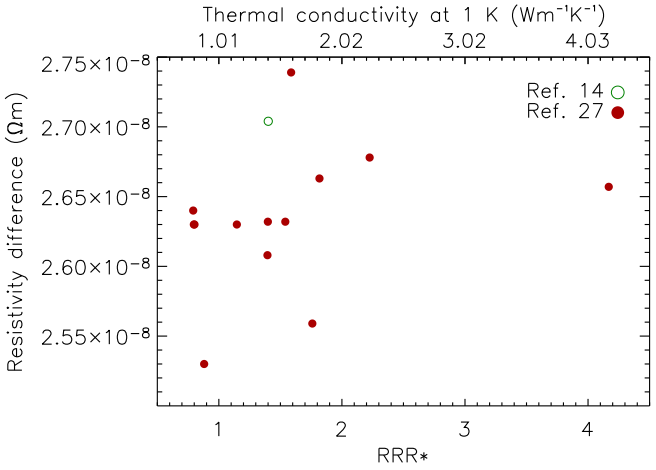

Figure 4: The difference between room temperature and residual resistivity for various aluminium alloy measurements in the literature [14, 27].

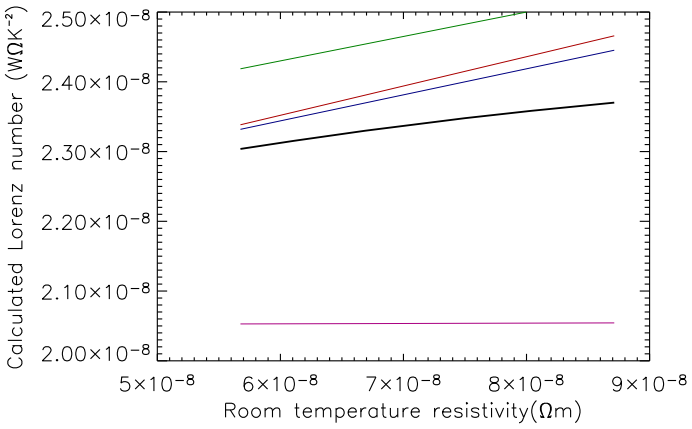

Figure 5: Prediction for the room temperature Lorenz ratio for aluminium alloys, as described in Section IV (thick line), compared with results from the literature [28] (thin lines).

is negligible at room temperature). Therefore, although there is no evidence of a variation of ρ_i with purity over the range shown in Figure 4, the value must change somewhere for $\mathrm{RRR}^* > 4$. This is not surprising since modifying thermal conductivity equations for pure aluminium to apply to alloys required altering the strength of the electron-phonon interaction.

It is therefore possible to take a single room temperature resistivity measurement (available for most aluminium alloys, and easily obtained otherwise) and derive an approximation to the thermal conductivity from room temperature down to the transition temperature. This method will fail for high purity alloys, since ρ_0 becomes so small compared with ρ_i that it cannot be measured. It should be valid for values of $\mathrm{RRR}^* < 4$, which is also the range over which the measurements in Figure 4 were taken. Most aluminium alloys fall into this range, with the commercially pure 1000 series alloys being the major alloys excluded.

We can now obtain a useful expression for RRR as a function of RRR^*. Using equations 8 and 13, we find

$$\mathrm{RRR} \simeq 1 + \mathrm{RRR}^* \frac{\rho_i}{\rho_{RT_{pure}}}, \qquad (14)$$

where, for low enough RRR^* ($\mathrm{RRR}^* < 4$ or $\mathrm{RRR} < 5.34$), the value of ρ_i can be taken from equation 13. This is convenient because RRR measurements are easier to make than resistivity measurements, since the geometrical factor of the sample does not have to be known.

It is also possible to use the methods described here to predict a value for the room temperature Lorenz number of aluminium alloys. This is done by taking a value for room temperature resistivity, extrapolating to the low temperature resistivity using Eq. 13, applying the Wiedemann-Franz law (Eq. 6), and extrapolating the resulting thermal conductivity to room temperature. The results are shown in Figure 5, where the prediction is compared with various values derived from measurements [28]. The predicted values fall within the range of experimental results.

V. MAKING PREDICTIONS

This section describes how the fit equations presented in previous sections can be usefully applied. Figure 6 shows the predicted thermal conductivity as a function of temperature for alloys of different purity. If the thermal conductivity of an alloy is known at a single temperature, the values at other temperatures can thus be extrapolated. As described in section IV, electrical measurements can also be used by converting them to thermal conductivity values using the Wiedemann-Franz law (Eq. 6).

However, there is a limit to the ability to make extrapolations from both thermal and electrical measurements. Figure 6 shows that the conductivity values converge with increasing temperature, and thus the accuracy of predictions will be highest for measurements made at low temperatures. This is shown more clearly in Figure 7, where the conductivity at 77 K and 300 K is plotted as a function of the conductivity at 1 K. A wide range of conductivity values at 1 K correspond to a very narrow range at 300 K. Figure 8 shows how the expected error in an extrapolated 1 K conductivity value varies with the conductivity at higher temperatures. For aluminium with a room temperature conductivity of approximately 210 $\mathrm{Wm}^{-1}\mathrm{K}^{-1}$ ($\mathrm{RRR}^* = 5.4$), a 1% error in a measurement at room temperature will cause a 10% error at 1 K. The size of this error increases rapidly with an increase in room temperature conductivity above 210 $\mathrm{Wm}^{-1}\mathrm{K}^{-1}$. However, as mentioned above, there are few aluminium alloys with such high values for the thermal conductivity.

Figure 9 shows a possible procedure for determining the thermal conductivity of a given aluminium alloy between the superconducting transition temperature and room temperature, depending on what information is available. It should

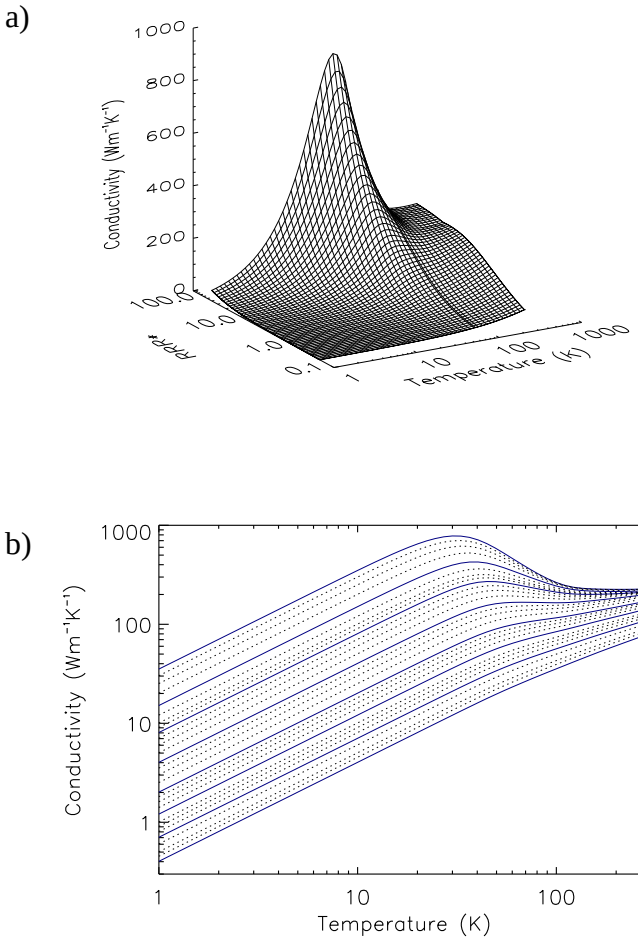

Figure 6: Predicted thermal conductivity values for aluminium alloys of different purity. These results can be used to extrapolate measurements made at one temperature on a given alloy to other temperatures. Fig. b) is a projection of Fig. a) into two dimensions. The solid lines in b) correspond to RRR^* values (Eq. 8) of 0.4, 0.7, 1.2, 2, 4, 8, 15 and 35; the conductivity increases with increasing RRR^*. Note that in a) the conductivity is given using a linear scale, while a logarithmic scale is used in b). These values apply to aluminium in the non-superconducting (normal) state; the superconducting transition temperature is above 1 K for some alloys (Section II). The graphs were produced using fits as described in Table II, row 9.

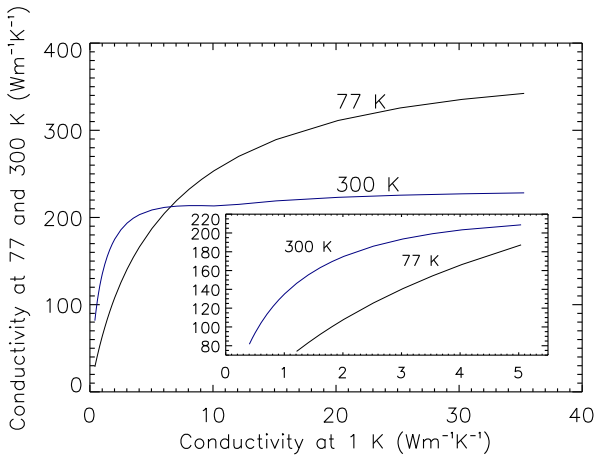

Figure 7: Conductivity of aluminium alloys at 77 and 300 K as a function of the 1 K value, using the predicted values shown in Fig. 6

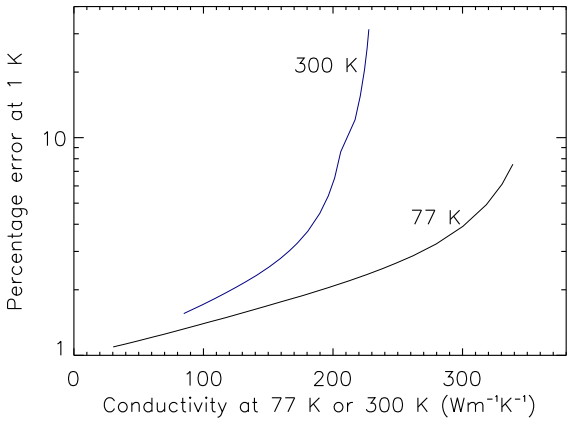

Figure 8: Percentage error in the extrapolated conductivity at 1 K for a 1% error in measured conductivity at 77 K and 300 K, using the predicted values shown in Fig. 6

be noted that if no useful measurements are available for a particular alloy type, room temperature electrical resistivity measurements can be made quite easily. However, if even that is not practical, it is possible to predict room temperature properties based on the composition of an alloy [28].

Fig. 9 refers only to measurements at low temperatures and room temperature; thermal (but not electrical) measurements at other temperatures, for example 77 K, may also be used. Again, the usefulness of a measurement depends on the alloy purity, but - as shown in Figs 7 and 8 - measurements at 77 K are useful for a wider range of purities than those made at 300 K.

There are, however, some potential problems. Firstly, room temperature conductivity values generally quoted for aluminium alloys may have significant errors. Making accurate thermal conductivity measurements near room temperature is difficult due to thermal radiation, and errors (especially systematic) are often underestimated. Moreover, the measurements quoted may not even have been intended to be of sufficient accuracy to be used in this manner; an error of 5 or 10% is acceptable in most applications. Therefore extrapolations from room temperature thermal conductivity measurements should be treated with extreme caution unless the details of the measurement are known (and preferably confirmed by measuring a standard reference material) and plausible errors quoted. Indeed, extrapolating manufacturer's quoted thermal conductivity values for the various alloys described here pro-

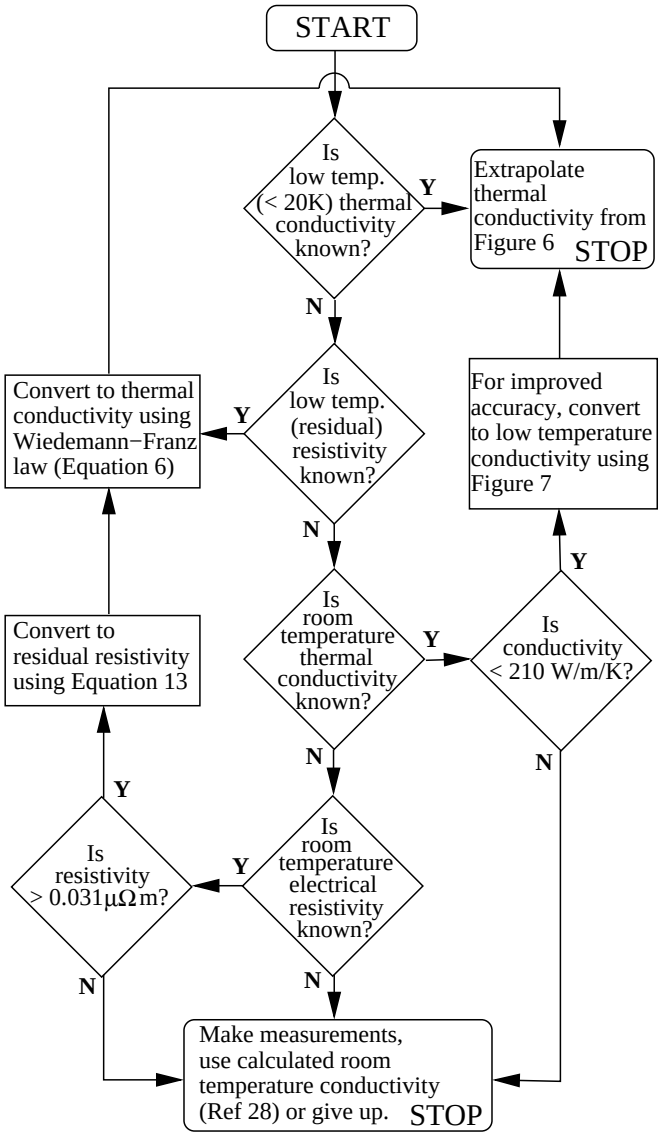

Figure 9: Procedure for predicting the thermal conductivity of an aluminium alloy depending on what information is available. More detail is given in Section V.

duces errors in the low temperature conductivity varying from approximately 4% to 50%. Using room temperature electrical resistivity measurements may provide a better prediction, since it is much easier to measure the resistance of an aluminium alloy sample at room temperature accurately than it is to measure the thermal conductivity.

The second problem is that there may be significant variation between samples of a given alloy and temper. For a given alloy type, a range in concentration of various impurities is usually allowed [30]. Variation within these ranges may cause considerable differences in thermal conductivity. Differences in tempering procedures may also cause significant variation, while still remaining within the specifications for the process. It is recognized that even at room temperature, different manufacturers quote different values for the resistivity of the same alloy type [28]. Another factor is that most thermal conductivity measurements were made several decades ago; changes in production methods may cause modern alloys to have different properties. Some large differences *are* apparent in the data presented here: values for 2024 T4 disagree by over 50% at 1 K, and values for 6061 T6 by over 30%. While such discrepancies may not be typical, and may to some extent be due to experimental error, errors due to sample variation may dominate the errors in the prediction methods described here.

Table V: Some residual resistivity measurements from the literature for aluminium alloys.

Alloy	ρ_0 ($\mu\Omega$m)	RRR*	1-K conductivity[a] (Wm^{-1}K^{-1})	Source
1100 O	8.20E-04	29.63	29.88	[27]
2014 T651	0.01531	1.59	1.60	[27]
2024 O	0.00583	4.17	4.20	[27]
2024 T4	0.01742	1.39	1.40	[27]
2024 T6	0.01337	1.82	1.83	[27]
2024 T86	0.01578	1.54	1.55	[27]
5083 H113	0.0305	0.80	0.81	[27]
5083 H113	0.0307	0.79	0.80	[27]
5083 O	0.0303	0.80	0.81	[27]
6061 T6	0.01381	1.76	1.77	[27]
6061 T6	0.009	2.7	2.72	[26]
6082 T6	—	2.70[b]	2.72	[29]
7039 O	0.0212	1.15	1.16	[27]
7039 T61	0.01738	1.40	1.41	[27]
7039 T61	0.01734	1.40	1.41	[14]
7075 T6	0.0276	0.88	0.89	[27]
7075 T73	0.01092	2.23	2.25	[27]

[a] This is the conductivity at 1 K *calculated* from the value of RRR* shown using the Wiedemann-Franz law (equation 6).
[b] Calculated from measured value of RRR using Eq. 14

Sample variation also affects the use of measured conductivity values from the literature. Another use for the equations presented here would be to predict the expected variation at low temperatures given a knowledge of the variation in room temperature properties, or to predict the thermal conductivity of each lot of material used in a particular system based on a simple electrical resistivity measurement.

Figure 10 shows recommended values for various alloys, based on low temperature thermal and electrical measurements from Tables III and V. For clarity, alloys with similar values have been grouped together. It should be noted that even at room temperature, samples of the same alloy but different tempers can have significantly different conductivity. This can be seen in Fig 10; good examples are aluminium 2024 O, T4 and T6. When using literature results it is therefore important to ensure that the temper is stated as well as the alloy type. Many measurements were excluded from Fig. 10

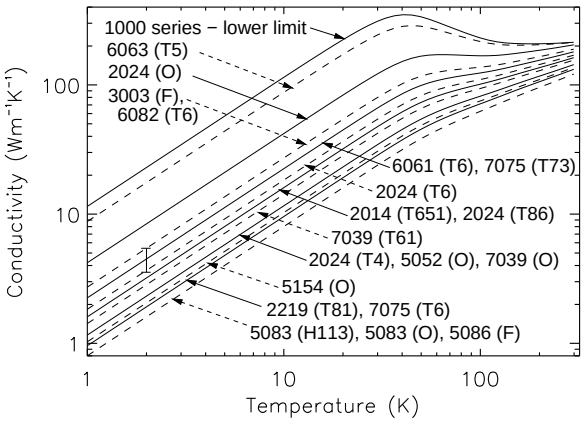

Figure 10: Recommended values for the thermal conductivity of various aluminium alloys, using the prediction method described in this paper on measurements from the literature listed in Tables III and V. For each alloy, the first four figures (e.g. 6061) indicate the alloy composition; the remainder (e.g. T6) indicates the temper. Note that for alloys 2024 (T4) and 6061 (T6), significantly different values are reported from different measurements; the mid-points of the ranges of values are shown here. An error bar shows the range of values obtained for 6061 T6. For clarity, similar values have been grouped together - the worst error resulting from this grouping is under 4%. For more accurate results, the RRR^* values quoted in this paper should be used to generate conductivity values using the equations in this paper or Fig. 6. The line styles alternate between solid and dashed curves to aid the eye. These results apply to aluminium in the non-superconducting (normal) state; the superconducting transition temperature is above 1 K for some alloys (Section II).

because the temper was not given.

VI. CONCLUSIONS

I have shown that it is possible to usefully predict the thermal conductivity of an aluminium alloy at temperatures between the superconducting transition temperature and room temperature, from a measurement at a single temperature.

The predictions are based on a set of semi-empirical equations for the thermal conductivity of *pure* aluminium [2]. They have been modified by altering the term describing the strength of the electron-phonon interaction from a constant to a function of the alloy purity. The modification was determined by examining various thermal conductivity measurements in the literature, and extends the lower limit of validity from approximately $\mathrm{RRR} = 10$ to $\mathrm{RRR} < 2$.

Extrapolating the thermal conductivity from low temperature measurements generally produces results which agree with true measured values by 10% or better. It is also possible to make extrapolations with a similar accuracy from room temperature measurements. However, this is only possible if the room temperature measurements are made with sufficiently good accuracy; this is not practical to achieve for the purest aluminium alloys but is otherwise possible.

Low temperature electrical resistivity measurements can also be used by applying the Wiedemann-Franz law to obtain the thermal conductivity, and then extrapolating to other temperatures. The Wiedemann-Franz law is not known with sufficient accuracy to be applied directly to room temperature electrical resistivity measurements, but can be applied indirectly by calculating the low temperature (residual) resistivity from the room temperature value and applying the Wiedemann-Franz law at low temperatures. This is possible because the difference between residual and room temperature resistivity is approximately constant for most (aluminium) alloys.

These equations allow aluminium alloys to be used in situations where the thermal conductivity is important, even if they are not one of the relatively few materials for which the conductivity has been measured down to low temperatures. Another important application is the ability to predict thermal conductivity from a single low temperature electrical resistivity value. This can be accurately measured much more easily than thermal conductivity. It is even practical to make such a measurement on a sample from each lot of aluminium alloy used in a cryogenic system, giving thermal conductivity values which are not affected by sample to sample variations. The equations can also be used as a check on the plausibility of thermal conductivity measurements, either obtained directly or found in the literature.

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