An introduction to semiconductor bolometer modelling -or- You too can be a bolometer expert Version 1.0

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1 Introduction

A common method to help characterise cryogenic semiconductor bolometers is to take "load curves" – measurements of the bolometer voltage as a function of current. This document explains what measurements are usually made, how they are used to obtain information on the bolometers, and mentions some common misconceptions. In particular my aim is to point out how simple modelling a bolometer in this way really is, even though this is not always evident. I do not intend to go into any area in great detail, since this is available elsewhere. Instead, I will concentrate on the general principles, some of which are not made particularly clear in the usual works. Further reading is suggested in Box 4.

The results from modelling can also be used to predict the noise performance of a bolometer [1]; this is somewhat more complex and is not discussed here. For completeness, footnotes give more detailed information on various topics, and for most purposes can be safely ignored.

2 Definitions and assumptions

We consider a bolometer to be a very simple device, shown schematically in Fig. 1. It consists of an absorber at temperature T which is weakly thermally linked to a heat sink at temperature T_0. We shall generally describe T_0 as the *stage* temperature, since when doing measurements it corresponds to the cold stage of the fridge we are using[1]. We

[1] In practise there will be a small thermal gradient between the thermometer on the fridge cold stage and the heat sink of the bolometer itself. This is generally too small to worry about.

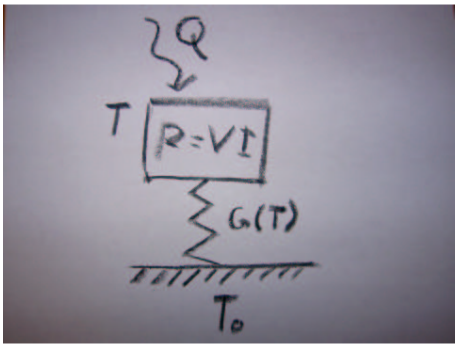

Figure 1: Ideal bolometer schematic

Box 1

Departures from the assumptions made in this document as often known as "non-ohmic" effects, and generally seem to fall into two types.
Electric field effects occur when the bolometer resistance is a function of voltage as well as temperature ($R = R(T, V)$).
Some bolometers also exhibit electron-phonon decoupling. This is an effect in which there is a finite thermal conductance between the electrons and the phonons in the resistor. Although not fully understood, it can be modelled well by treating them as physically separate components separated by a thermal conductance which varies as a power-law with temperature.
If non-ohmic effects are present, then many of the conclusions in this document are not true. However, the ideal bolometer model can be extended to include these effects. For results obtained without using the model, it is often possible to find areas in which these effects are negligible and to still obtain valid results.

usually call the absorber temperature, T, the *bolometer* temperature. We assume that we can write the thermal conductance of the weak thermal link as $G(T)$ (note that is is a function of temperature). This is usually a very good assumption, unless electron-phonon decoupling is present (see Box 1) [2].

Attached to the absorber is a resistor; we use the fact that the resistance changes with temperature to determine the absorber temperature. We assume that we can write the resistance as $R(T)$; in other words, it depends on temperature alone. This doesn't have to be the case (see Box 1). There are two sources of power dissipated in the bolometer absorber. Electrical power from the resistor, P, is just VI where V is the voltage across the bolometer and I is the current through it. There may also be incident "optical" (sub-mm) power, which we call Q here. We also make the assumption that both types of power have exactly the same effect on the bolometer[3].

As well as the assumptions described above, the *ideal thermal model* assumes the

[2] It is not necessarily the case; while any single material will have a conductance of the form $G(T)$, the link may be formed by two sections in series or parallel. Thermal conductance can also depend on quantities other than temperature, such as magnetic fields.

[3] This need not be the case if electron-phonon decoupling is present (Box 1), since the electrical power heats the electrons in the thermometer, and optical power heats the lattice.

Box 2 *List of terms*

Term	Definition	Alternative name	Section
T	bolometer absorber temperature	bolometer temperature	2
T_0	bolometer heat sink temperature	stage temperature	2
$G(T)$	thermal conductance of bolometer thermal link		2
$R(T)$	bolometer resistor resistance	bolometer resistance	2
P	bolometer resistor electrical power		2
V	voltage across bolometer resistor		2
I	current through bolometer resistor		2
G_o	Thermal model parameter (eq. 1)		2
β	Thermal model parameter (eq. 1)		2
R^*	Thermal model parameter (eq. 2)	R_0	2
T_g*	Thermal model parameter (eq. 2)	Δ, T_0	2
m	Thermal model parameter (eq. 2)		2
R_0	Zero bias resistance		3.1
Q	optical power absorbed by bolometer		4
T'	Equivalent stage temperature for optical power Q		4.2

following:

– The thermal conductance follows a powerlaw, i.e.

$$G(T) = G_o T^\beta \tag{1}$$

– The thermistor resistance follows the following equation:

$$R = R^* \exp\left(\left(\frac{T_g}{T}\right)^m\right) \tag{2}$$

These assumptions are both based on physical principles, and in practise are generally appropriate. The parameters G_o, β, R^*, T_g and m are constant, and the aim is to determine these parameters for a given bolometer. We can then use the model to predict the bolometer voltage for any bias current and absorbed optical power. Determining G_o is also useful because it is usually a design goal.

It is usually assumed that $m = 0.5$. This is not necessarily so but is often a good assumption [2].

Note that not everybody uses the same terminology for the resistance: R^* is sometimes called R_0, and T_g may be called Δ or T_0. A list of terms used in this document is given in Box 2.

3 Using the model

The game with the ideal bolometer model is to fit "load curves"; these are measurements of the bolometer[4] voltage as a function of current. Generally we make such measurements

[4]To be pedantic, they are measurements of the bolometer *resistor* voltage.

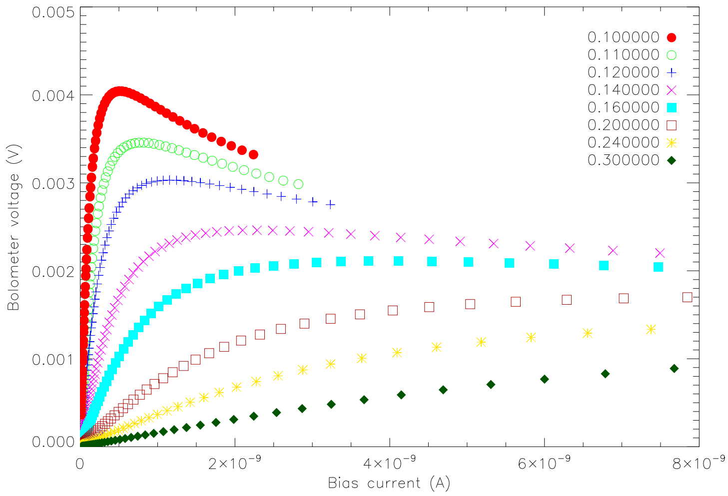

Figure 2: An example of load curve measurements. Different symbols correspond to different stage temperatures as shown (the bolometer is HFI CQM 100 GHz SWB S/N 07).

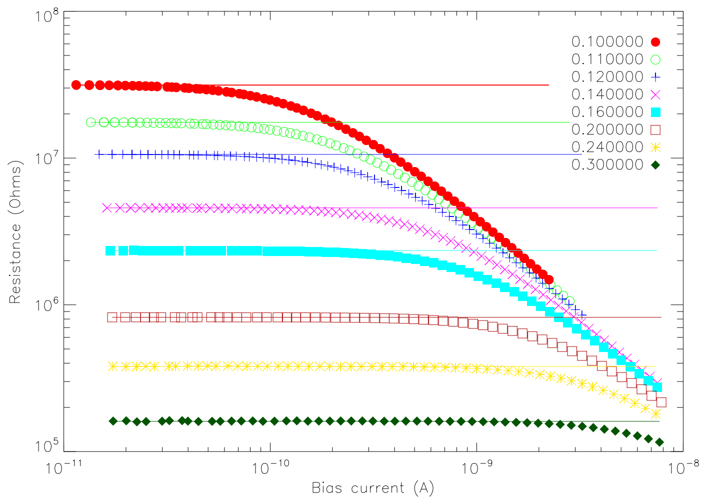

Figure 3: The load curves from Fig. 2, plotted as current vs resistance. The solid lines show the chosen values for the zero bias resistance.

at various stage temperatures, to obtain a family of load curves. An example is shown in Fig. 2. For clarity, not all the measurements are shown.

3.1 R(T)

There are two stages to using the model. The first is to determine $R(T)$, using equation 2. Since we have plotted V as a function of I, we can obtain R from the gradient of the plots in Figure 2. However, it is clear that the gradient is not constant, and indeed becomes negative for some of the measurements. This is because as the bias current increases, the

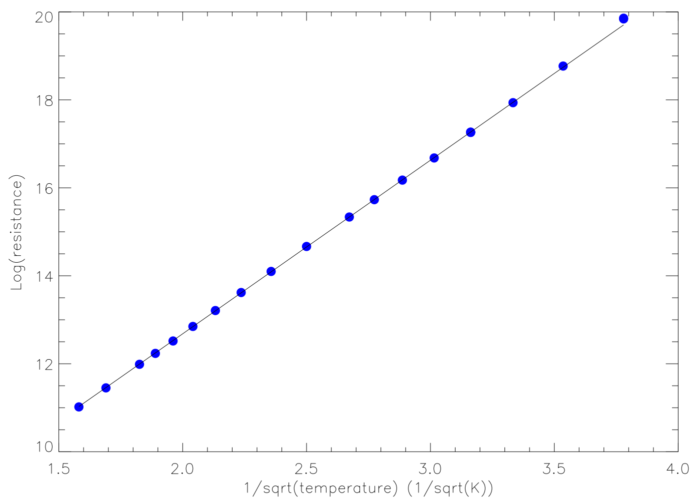

Figure 4: Plot used to determine bolometer resistance as a function of temperature. Solid points are measured data; the line is a least squares fit.

bolometer heats up, reducing the resistance. This complicates using the bolometer, but at this stage it just means that we have to make sure we use the start of each load curve where the resistance is constant. It does mean we have to use our judgement on how much of the load curve to use. Most people seem to prefer to look at V as a function of I to spot the point at which the curve deviates from a straight line. I prefer to plot $R = V/I$ on a log scale, since it makes it much easier to see how much of the load curve we can safely use; an example is shown in Fig. 3.

We can now plot the zero bias resistance (R_0) as a function of temperature. If we plot $\log(R_0)$ as a function of $1/\sqrt{T_0}$ (Fig. 4) then the gradient and offset of a linear fit give us T_g and R^*, assuming that $m = 0.5$. We should always look at the fit residuals to see that this value of m is applicable. If not, we can do a non-linear fit to obtain the best fit values of m, R^* and T_g. Most people just look at the results in the form of Fig. 4; however, it is much harder to see departures from $m = 0.5$ [2].

In principle all thermistors diced from the same wafer should have the same value of T_g, though in practise one sees some variation. The value of R^* will depend on the dimensions. Note that since it is $log(R^*)$ that appears in equation 2, relatively large changes in R^* have little effect. The value of R^* also depends strongly on the fitted value of T_g.

3.2 G(T) - method 1

Now we just have to find G_o and β. This is the point at which the equations can start to look complicated. But what's going on is very simple. We are just measuring the thermal conductance of a "sample" (Fig. 5), just as we might measure the thermal conductance of, say, a piece of Torlon® to be used in an instrument. We have one end which is heat sunk at a chosen temperature (T_0); the other end has a heater and a thermometer. We heat up one end, and can calculate the conductance from the temperature rise and power applied. The only difference here is that instead of a separate heater and thermometer, we use the bias heating from the thermometer to heat the sample.

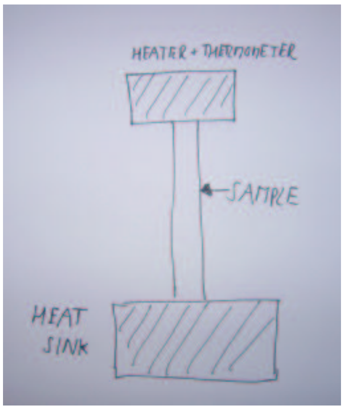

Figure 5: Layout used to measure the thermal conductance of a material sample.

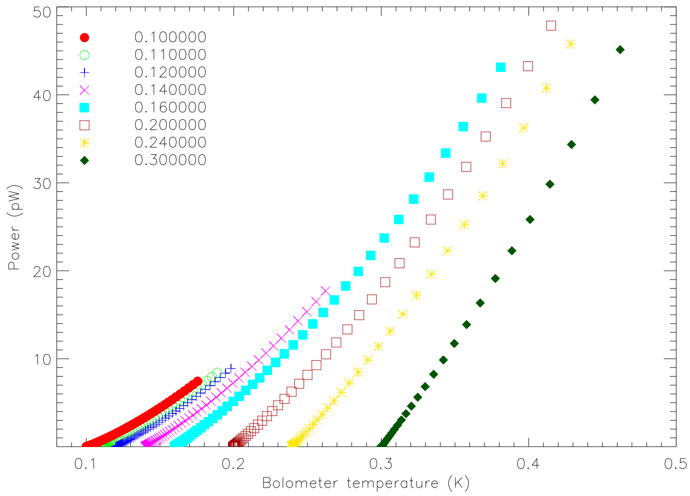

Figure 6: Measurements from Fig. 2, plotted as power vs temperature.

If the thermal conductance, G, was constant, then we could simply write (by definition):

$$P = G\Delta T, \tag{6}$$

where ΔT is the temperature difference across our sample. More generally, we can write

$$P = \int_{T_0}^{T} G.dT. \tag{7}$$

Since we have assumed equation 1, we can write this[5] as

$$P = \frac{G_o}{\beta + 1}\left(T^{\beta+1} - T_0^{\beta+1}\right). \tag{8}$$

We can now do a non-linear fit to P as a function of T to obtain both G_o and β (see Fig. 6). We can also include T_0 as a fit parameter if we aren't sure of our stage temperatures.

[5]This is not the form most people use – see Box 3.

Box 3

It is customary to replace equation 1 with

$$G = G_{so} \left(\frac{T}{T_0} \right)^{\beta}. \tag{3}$$

We then find that

$$G_o = \frac{G_{so}}{T_0^{\beta}} \tag{4}$$

and

$$P = \frac{G_{so} T_0}{\beta + 1} \left(\left(\frac{T}{T_0} \right)^{\beta + 1} - 1 \right). \tag{5}$$

I don't find this especially helpful as one then has to allow for G_{so} being a function of T_0, which can be a nuisance, and I believe it also makes the equations look unnecessarily more complicated. Likewise people sometimes work with the parameter $\phi = T/T_0$ instead of T.

Now, to do this, we don't need to worry about the fact that the heat is actually coming from the resistor we are using to measure temperature. Power is just given by $P = VI$, and we can measure both V and I. Likewise, the temperature is just given from the resistance ($R = V/I$) and equation 2. It is customary to plot the results from fits using equation 8 in terms of current and volts, rather than power and temperature, but the fitting is still ultimately to power vs temperature[6].

Thinking of the fitting in terms of voltage and current can lead to misconceptions, with probably the most popular being that in order to constrain β and G_o, it is necessary for the data to include the "downturn" in the load curves (see Fig. 2). Since there is no downturn in a plot of power vs temperature, it can't matter at all. What we do need is a large enough current to make the bolometer temperature cover a useful range.

3.3 Applying the model

So now we have the parameters R^*, T_g, m, G_o and β. Assuming that the model produced good fits, we can now predict the bolometer voltage as a function of current. We start with an appropriate temperature range, and use equation 8 to give the corresponding powers.

[6] In a similar manner, if doing a conventional thermal conductance measurement as in Fig. 5 using a separate heater and thermometer, we *could* plot the results in terms of the current through the heater and voltage across the resistor measuring temperature. But we probably wouldn't.

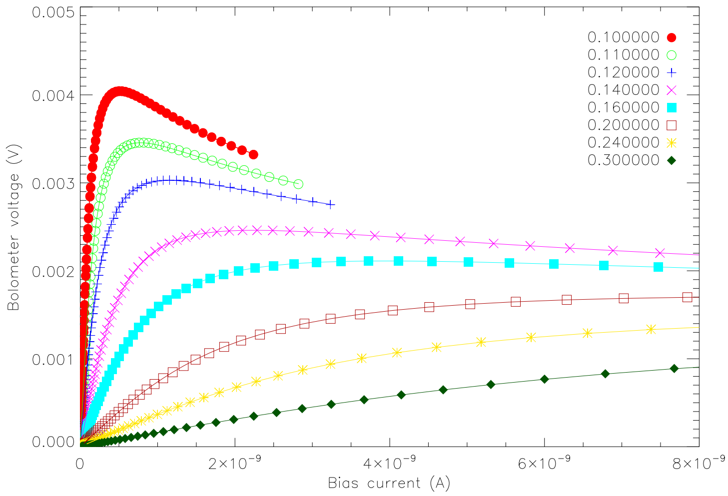

Figure 7: Measurements from Fig. 2, also showing fits using the thermal model.

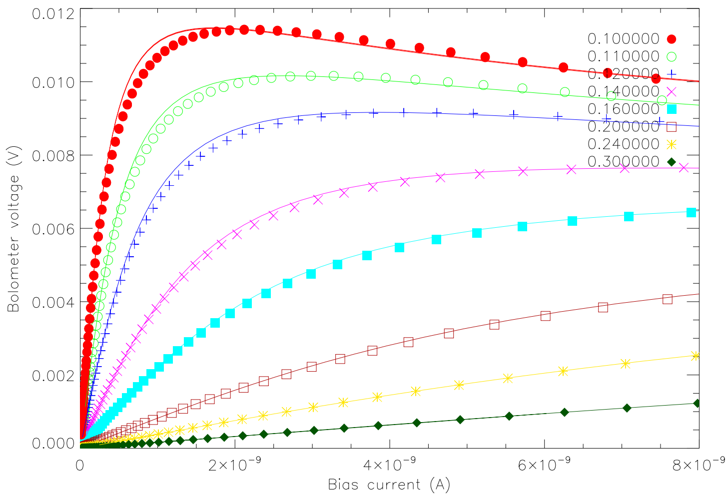

Figure 8: Measured load curves and thermal model fits for another bolometer (HFI CQM 857 GHz SWB S/N 01).

We can then get resistance from equation 2, enabling us to calculate[7]

$$V = \sqrt{PR} \tag{10}$$

[7]Putting equations 10, 8 and 2 together, we can write

$$V = \sqrt{\frac{G_o}{\beta+1}\left(T^{\beta+1} - T_0^{\beta+1}\right) R^* \exp\left(\left(\frac{T_g}{T}\right)^m\right)}, \tag{9}$$

which makes the model look terribly complex. However, since we have determined R^* and T_g already, these aren't fit parameters, and we might as well remove them from the equation before carrying out the fit.

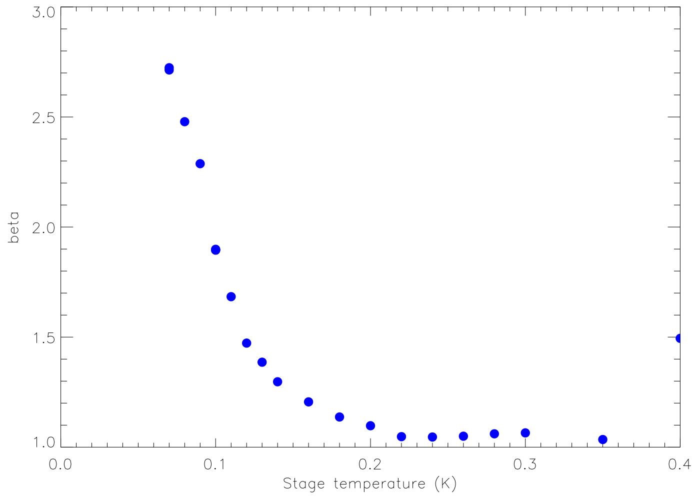

Figure 9: Values of the thermal model parameter β as a function of stage temperature for the data in Fig 8.

and

$$I = \sqrt{\frac{P}{R}}. \tag{11}$$

Note that since this is a parametric equation, if we want voltage measurements at particular values of current, rather than the values that we end up with for the chosen values of T, we'll have to either interpolate, or use an iterative technique. This is rarely necessary, however.

The fits in Fig. 7 show very good agreement with the measurements. This is not always the case. The culprits are usually non-ohmic effects (Box 1).

There is a subtlety though. It is usual to carry out separate thermal model fits to each load curve. One can then find good looking fits, but with values of β that vary systematically by a significant amount from load curve to load curve. This is not in agreement with the thermal model, since β is supposed to be a constant. An (extreme) example is given in Figs 8 & 9. While the fits look good, the value of β varies wildly with temperature. This is telling us that the thermal model is not working well[8]. We should therefore be very cautious about applying the thermal model, and indeed for temperatures other than those measured, it isn't even obvious what value of β we should adopt. We need to use a more sophisticated model and attempt to include either electric field effects or electron-phonon decoupling and see if these improve matters.

An alternative approach to the conventional two-step method outlined here would be to fit each load curve with R^* and T_g as adjustable parameters as well as G_o and β. It is impractical to constrain these parameters well enough from a single load curve [3]. However, it can be done if a fit is carried out simultaneously to a set of load curves. This method has the advantage of not requiring any judgement as to how to determine the zero bias resistance. However, it will suffer if non-ohmic effects are present, unless they are also effectively included via extra fitting parameters. This method might even be useful if

[8] Another clue is that β turns out to be a function of the bias range over which the bolometer is measured

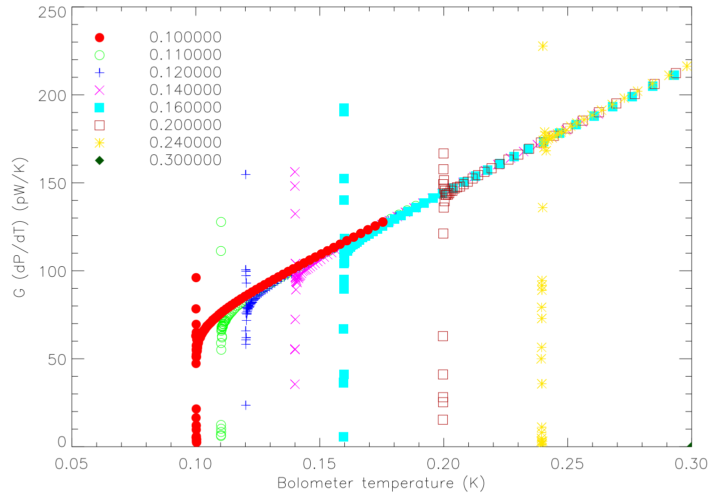

Figure 10: Measurements from Fig 2, plotted as $G = dP/dT$ as a function of bolometer temperature.

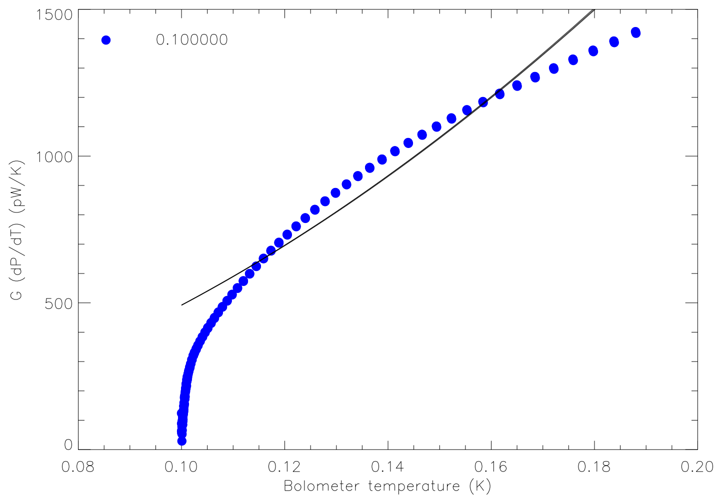

Figure 11: 100 mK load curve from Fig. 8 and thermal model fit, plotted in $G(T)$ space.

the optical power is unknown.

3.4 G(T) - method 2

The methods described above are the traditional way to model bolometers. However, there is an alternative method of deriving $G(T)$, so long as you are prepared to do a little numerical differentiation. We can re-write equation 7 as

$$G = \frac{dP}{dT}. \tag{12}$$

We can then plot G as a function of T for each load curve, and obtain G_o and β from a simple power-law fit. As always, one has to be careful when differentiating real data, and some smoothing to remove noise may be necessary. Another problem is that the noise will blow up for the section where T is barely changing because the bias power isn't large enough to cause significant self-heating. It is therefore necessary to exclude this part of the load curve from the fits. For this reason, it can be easier to do fits using the traditional method.

However, it is still very useful to plot the data in this form, since it is an excellent way of comparing different load curves. Since $G(T)$ depends only on the bolometer temperature, measurements from different load curves should fall on the same curve. Deviations tell you either that the model isn't working properly, or that there is a problem with the measurements. Generally, failures of the model (electric field effects or electron phonon decoupling) manifest themselves as a down-turn of $G(T)$ at the lower temperature end of each load curve. Other discrepancies are usually measurement problems. An example is shown in Fig. 10. From this it can be seen instantly that the different load curves are generally in very good agreement, but there is a hint of deviation at the lower temperature end of each load curve for the lower stage temperatures. Compare this to Fig. 2, where it would take a very trained eye to be able to see if the different load curves were consistent with each other.

The usefulness doesn't end here. The 100 mK load curve from Fig 8 looks as if the thermal model fit is quite good. But if the same data and fit is plotted as $G(T)$ (Fig. 11), we can see that the fit is actually very poor, even though it looked quite good when viewed in voltage-current space. We should therefore treat anything obtained from the model for this bolometer with suspicion. In particular, the value of G_o is not going to be the true thermal conductance.

This is also a very useful method of comparing load curves taken with different (and even unknown) optical power. Since we are plotting $G = dP/dT$, any optical power is constant and therefore doesn't affect the value we obtain. So a group of load curves taken at arbitrary optical power and stage temperature should all overlap. This is particularly useful when comparing load curves taken in two different systems, since they differences in thermometry are likely to mean that even if an attempt is made to make measurements at the same temperatures, this will not be the case. With this method, not only do we not need measurements at the same temperatures, we don't even need to know what the temperatures are[9].

4 Optical power

So far we have only touched on what happens with optical power present. Obviously our modelling is not going to be very useful unless we can include this. This section discusses how the various methods above are affected by the presence of optical power.

4.1 R(T)

Any optical power will raise the bolometer temperature above the heat sink temperature even at zero bias. Therefore it becomes much harder to determine R(T), and it is best to use measurements taken with negligible optical power. Since the effect of power will

[9] We have to believe the temperatures in one system in order to derive $R(T)$, though

depend on G, which in turn depends on stage temperature, in principle it would be possible to obtain an expression for $R(T)$ by including the optical power as a fit parameter. In practise it is not easy to get accurate values this way.

4.2 G(T)

As has been mentioned before, adding optical power makes no difference to values for $G(T)$, so plots of $G(T) = dP/dT$ can be used as before. If using fits to the thermal model, the addition of optical power, Q, changes equation 7 to

$$P = \int_{T_0}^{T} G.dT - Q, \tag{13}$$

and equation 8 to

$$P = \frac{G_o}{\beta+1}\left(T^{\beta+1} - T_0^{\beta+1}\right) - Q. \tag{14}$$

We can then do a non-linear fit including Q can as a fit parameter. However, we must then supply a value for T_0 – we can't include it as a fit parameter, since T_0 and Q are degenerate. In fact it is a basic principle that an optical load has exactly the same effect as a change in stage temperature[10].

We can see this if we define T' such that

$$Q = \int_{T_0}^{T'} G(T).dT. \tag{15}$$

Then we can rewrite equation 13 as

$$Q + P = \int_{T_0}^{T} G(T).dT = \int_{T_0}^{T'} G(T).dT + \int_{T'}^{T} G(T).dT \tag{16}$$

and thus

$$P = \int_{T'}^{T} G(T).dT. \tag{17}$$

Comparing with equation 7, we can see that a finite Q is just the same as $Q = 0$ and T replaced with T'. Once consequence of this is that we can obtain a value for Q from the bolometer temperature at zero bias without needing a full load curve. Note that this conclusion does not depend on $G(T)$ having the power-law form assumed in the ideal thermal model.

Within the thermal model, we can then rewrite equation 15 as

$$Q = \frac{G_o}{\beta+1}\left(T'^{\beta+1} - T_0^{\beta+1}\right), \tag{18}$$

and thus

$$T' = \left(\frac{(\beta+1)\,Q}{G_o} + T_0^{\beta+1}\right)^{\frac{1}{\beta+1}}. \tag{19}$$

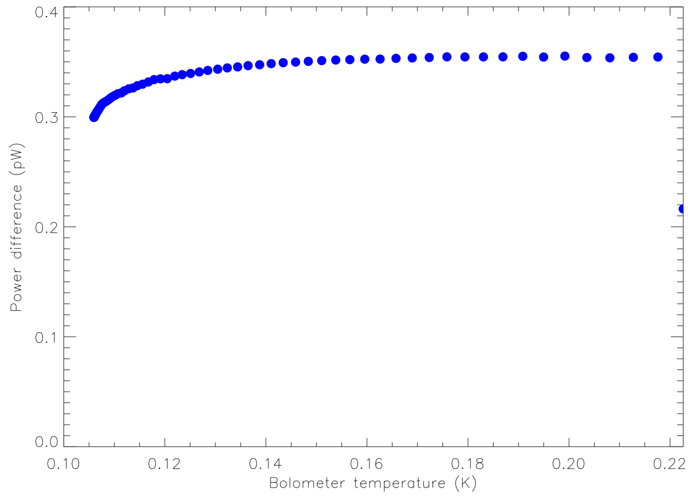

Figure 12: Difference in optical power for load curves taken with 77 K and 300 K blackbody loads at 100 mK stage temperature (the bolometer is HFI CQM 100 GHz SWB S/N 08).

4.3 ΔP

There is a further method of obtaining the optical power from a load curve, without using the thermal model. This method can only determine the *difference* between optical power for two load curves, but is nevertheless very useful.

This method relies on the fact that the bolometer temperature is a function of the bolometer power alone. So if we have two load curves taken at the same stage temperature, and we pick a point on each curve at the same temperature (and thus resistance – we don't even need to know the form of $R(T)$), then the total power ($P + Q$) must be the same. Since we know $P = VI$ for each load curve, we can easily calculate the value of Q. This method will give a value of Q as a function of P. In principle, of course, the value of Q should be constant. In practise small systematic errors will prevent this from being the case, and it is necessary to use some judgement to choose a value for Q. An example is shown in Figure 12. Agreement seems to be better for measurements made during the same cool-down. The great advantage of this method is that it does not require the thermal model to hold, or any of the thermal model parameters to be known.

Note that if we fail to have the two load curves at the same temperature, the value of Q should still be constant, but will be incorrect. If we know G_o and β, and have accurate values for the temperatures, we can correct the value of Q using equation 18.

5 Applying these methods

This section briefly discusses what measurements should be made, and how to interpret them. When building bolometers for an instrument, there are often (at least) two stages:
– characterisation, where the bolometers are measured in some kind of test cryostat

[10] So long as electric field effects and electron-phonon decoupling are negligible.

– a second stage, sometimes referred to as calibration, where they are tested in the instrument they were intended for.

This second stage is likely to be carried out using read-outs optimised for doing observations rather than characterisation – for example it may be an AC readout, whereas the characterisation may have been done using a more straightforward DC readout.

5.1 Characterisation

At this stage the game is usually to make sufficient load curve measurements to obtain the thermal model parameters. This enables us to know if the goals have been met. These normally include specified values for:

– G_o, itself a model parameter
– Optical efficiency – this can be determined from measurements with different optical loads.
– Detector noise – a value for this can be derived from the thermal model parameters, though this is not discussed here.
– Speed of response – this requires separate measurements and will not be discussed here either.

So we need a cryostat which can make measurements in a near negligible optical background, at various stage temperatures. We call these *blanked* measurements. The minimum set of blanked measurements required to obtain the thermal model parameters is:

– Load curves at many different temperatures in order to determine $R(T)$. Since we only need the zero bias value, these do not need to go over a large bias range, though it is useful if they do.
– At least one load curve over a large enough bias range so that we can obtain G_o and β.

However, if we make all the load curves over a large bias range, this helps ensure the accuracy of our measurements by looking at the consistency of results. If all is well, we should obtain the same value of G_o for each load curve, and plots of $G(T)$ should overlap as discussed earlier.

To obtain optical efficiency, we make measurements with different optical powers, and calculate Q using one (or ideally more than one) of the methods described above. If we can then calculate the incident power on the bolometer (or, more usually, the optical chain), we can calculate the optical efficiency from the value of Q.

5.2 Calibration

In this stage, the bolometers are re-measured in their instrument. Here the goals are slightly different, and may include:

– Ensuring that the readout system is well understood
– Ensuring that thermal model parameters determined earlier can be used to predict the bolometer performance
– Measuring the optical efficiency of the bolometers in the instrument

To ensure that the readout system is behaving as expected, we measure load curves and see if they agree with those taken during characterisation. If not, there is a problem! Since there may well be doubt over the absolute optical power and even stage temperature, comparing $G(T) = dP/dT$ is a very useful technique. It is also useful to be able to show

Box 4 *Suggested further reading*

– Sudiwala, Griffin and Woodcraft [3] gives a far more detailed description of the thermal model than is presented here.
– Grannan, Richards and Hase [1] extends the model to allow for electric field effects in the thermistor.
– Richards [4] and Jones [5] both describe various aspects of bolometer operation and modelling.
– Woodcraft et al. [6] demonstrates the application of the thermal model by using it to characterize a bolometer designed for 100 mK operation.

that the thermal model parameters determined during the characterisation stage can be used to predict the bolometer performance in the instrument, since it is usually impossible to make the required measurements to re-characterise the bolometers in an instrument.

Optical efficiency is measured as before. This can be the most important measurement – even if it turns out that differences in the readout system mean that the load curves do *not* look consistent with those during characterization, the chances are that, although not ideal, this can be lived with. Ultimately, calibration will be done against optical sources, not using the bolometer model, although having the model is very useful. But optical efficiency *is* very important in demonstrating that an instrument will have acceptable performance.

6 And finally...

If all this made sense, and you want to learn more, then you could try the papers listed in Box 4.

References

[1] S. M. Grannan, P. L. Richards, and M. K. Hase. Numerical optimization of bolometric infrared detectors including optical loading, amplifier noise, and electrical nonlinearities. *Int. J. Inf. Mill. Waves*, 18(2):319–340, 1997.

[2] A. L. Woodcraft, R. V. Sudiwala, E. Wakui, and C. Paine. Hopping conduction in NTD germanium: comparison between measurement and theory. *Journal of Low Temperature Physics*, 134(3):925–944, 2004.

[3] R. V. Sudiwala, M. J. Griffin, and A. L. Woodcraft. Thermal modelling and characterisation of semiconductor bolometers. *Int. J. Inf. Mill. Waves*, 23(4):545–573, 2002.

[4] P. L. Richards. Bolometers for infrared and millimeter waves. *J. Appl. Phys.*, 76(1):1–24, 1994.

[5] R.Clark Jones. The general theory of bolometer performance. *Journal of the Opcial Society of America*, 43:1–14, 1953.

[6] A. L. Woodcraft, R. V. Sudiwala, M. J. Griffin, E. Wakui, B. Maffei, C. E. Tucker, C. V. Haynes, F. Gannaway, P. A. R. Ade, J. J. Bock, A. D. Turner, S. Sethuraman, and J. W. Beeman. High precision characterisation of semiconductor bolometers. *Int. J. Inf. Mill. Waves*, 23(4):575–595, 2002.