A note on the meaning of conductance and conductivity in bolometer analysis

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Contents

1 Introduction

I believe that descriptions of semiconductor bolometer modelling often make things look more complicated than they actually are, and can be somewhat confusing. In particular, the concept of "static conductance" is sometimes introduced. I believe that this serves no useful purpose; in this document I attempt to explain why.

2 Conductance, conductivity and power-laws

For a bolometer, we define total electrical power P by

$$P = VI, \tag{1}$$

where V is the voltage across the bolometer thermistor, and I is the current passing through it. The bolometer absorber temperature is T, and the heat sink (or "stage") temperature is T_0.

In bolometer analysis, a quantity called the static thermal conductance, G_s, is sometimes introduced. It is defined by the following equation:

$$P = G_s \left(T - T_0\right). \tag{2}$$

In general, G_s will vary with temperature (both T *and* T_0); if we assume it follows a power-law variation with T:

$$G_s(T) = G_{s0}T^{\beta}, \tag{3}$$

we have the Griffin and Holland model [1], which is wrong, since equation (3) is only true in general (if at all) when $T \simeq T_0$.

However, it is often the case that the thermal *conductivity* of the material forming the thermal link between the bolometer absorber and heat sink can be taken to follow a power-law. We can then write[1]

$$\kappa(T) = \kappa_0 T^{\beta}, \tag{4}$$

where $\kappa(T)$ is the thermal conductivity at temperature T. If we base our model on this, we have the Mather model, as used in references [2, 3].

However, we can't base the model directly on conductivity, since we also need to take the geometry of the thermal link into account. Assuming (without loss of generality) that the link has constant cross section A and length l, then we can define a quantity

$$G_d(T) = \frac{A}{l}\kappa(T), \tag{5}$$

and therefore from equation (4) we can write

$$G_d(T) = G_{d0}T^{\beta}. \tag{6}$$

In order to express power, P, in terms of G_d, we need to integrate $G_d(T)$ over the temperature range from T_0 to T:

$$P(T, T_0) = \int_{T_0}^{T} G_d(T)dT = G_{d0}\int_{T_0}^{T} T^{\beta} = \frac{G_{d0}}{\beta + 1}\left(T^{\beta+1} - T_0^{\beta+1}\right). \tag{7}$$

The quantity G_d is referred to in bolometer analysis as the dynamic thermal conductivity, presumably by analogy with dynamic (electrical) impedance. However, I have only ever come across the terms static and dynamic conductivity in bolometer analysis (try a web search on "static thermal conductance" or "dynamic thermal conductance" and note how almost all you find are pages on bolometer analysis).

There seems to be quite a lot of confusion in this area, which I believe results from lack of appropriate terminology. The difference between models based on equation (3) and (6) is sometimes said to be that in the former the conductance follows a power-law, but in the latter it is conductivity that follows a power-law. However, the real difference between condctivity and conductance is that conductivity is an intrinsic property of a material, and conductance is a property of a given thermal link with a particular geometry. In fact, although we introduced

[1] κ_0 is the conductivity at a temperature of 1 K; if we would like to quote κ_0 at temperature T_{ref}, we can write $\kappa(T) = \kappa_0 \left(\frac{T}{T_{ref}}\right)^{\beta}$ instead.

conductivity, κ, above in equation (4), by the time we reach equation (7), we have dropped conductivity in favour of conductance again, and we could have carried out the derivation without ever introducing it.

The problem seems to arise because the term conductance is used to describe both the ratio of temperature difference to power:

$$G_s(T, T_0) = \frac{P}{T - T_o}, \tag{8}$$

and to describe a property of a thermal link at a given temperature, so that

$$G_d(T) = \frac{dP}{dT}. \tag{9}$$

In the terminology of static and dynamic conductance, it would be much clearer to describe the models as differing by whether it is static (equation (8)) or dynamic (equation (9)) conductance that follows a power-law.

3 Static thermal conductance

Now, if we assume that equation (6) is valid, we can obtain an expression for static conductance. From equations (2) and (7), we find:

$$P = G_s\left(T - T_0\right) = \frac{G_{d0}}{\beta + 1}\left(T^{\beta+1} - T_0^{\beta+1}\right) \tag{10}$$

and therefore

$$G_s(T, T_0) = \frac{G_{d0}}{\beta + 1}\frac{T^{\beta+1} - T_0^{\beta+1}}{(T - T_0)}. \tag{11}$$

But G_s just consists of two separate parts:

$$G_s(T, T_0) = \frac{G_{d0}}{\beta + 1}\left(T^{\beta+1} - T_0^{\beta+1}\right) \times \frac{1}{(T - T_0)}, \tag{12}$$

where the left hand side does the actual "work" of integrating G_{d0} over the temperature range from T_0 to T, and the right hand side cancels out the unphysical $T - T_0$ term from equation (2). As such, it does not seem at all useful to me. In fact in bolometer analysis, I have only ever seen it used in the form of G_{s0}, defined as

$$G_{s0} = G_s(T_0 \to T, T_0) \tag{13}$$

for some reference temperature T_0. However, taking the limit $T \to T_0$ in equation (11), we find that

$$G_{s0} = G_{d0}, \tag{14}$$

so we could happily replace G_{s0} everywhere[2] with G_{d0} and never define static conductance in the first place.

[2]It is commonly used in place of G_{d0} in equation (7).

Outside the world of bolometer analysis, G_d is generally just referred to as conductance, G. The *only* use I can think of for G_s is to describe the basis of the incorrect thermal model in which G_s is taken to depend on temperature with a power-law. I therefore think that a much more useful and straightforward way of deriving the bolometer equations is to work entirely with G, as I do in my note on bolometer modelling [4].

Finally, I should note that it is common to make the above derivations look even more complicated by replacing T with $\phi = \frac{T}{T_0}$, which does nothing for the simplicity of the equations.

4 Two final comments

It is common to write equation (7) with G_{d0} (usually written as the equivalent G_{s0}) defined as the conductance at the heat sink temperature T_0; the value of G_{d0} is thus different for load curves taken at different stage temperatures. Alternatively, G_{d0} can be taken to be the conductance at a fixed temperature (usually the nominal operating temperature of the bolometer). Both approaches are equivalent, but can lead to confusion when comparing values obtained using the two different methods.

It has no relevance to the above, but while I'm here I'd like to point out that a common misconception is that in order to fit the thermal model to a bolometer load curve, it is necessary to include the "downturn" in voltage vs current. This is not true! The fits are actually carried out in temperature-power space, where there is no downturn. Obviously it is necessary to have a large enough temperature range to carry out a good fit, but it makes no difference whether the downturn is present or not when the data is viewed in current-voltage space.

5 Summary

I have asserted the following:

- The terms static and dynamic conductance appear to be unique to bolometer analysis.
- The concept of static conductance contributes nothing. In valid models, it only appears in the form G_{s0}, which is equivalent to G_{d0}. It should therefore be dropped in descriptions of bolometer modelling.
- Instead, dynamic conductance should be used everywhere, in which case it can be referred to simply as conductance, G, as is generally the case in areas other than bolometer analysis.
- An incorrect version of the thermal model has been used historically in which static, rather than dynamic conductance has been taken as a powerlaw. This is often referred to as a model in which conductance rather than conductivity is taken as a powerlaw. A better way to describe the two models is whether static or dynamic conductance follows a powerlaw[3].

[3]This is the only case in which the concept of static conductance seems to have a use - to describe the basis of a flawed model, which was based upon the unnecessary concept of static conductance

References

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