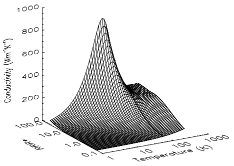

PREDICTING THE THERMAL CONDUCTIVITY OF ALUMINIUM ALLOYS AT CRYOGENIC TEMPERATURES

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1 Introduction

It is often important to know the conductivity of aluminium alloys at cryogenic temperatures. However, there are many types of aluminium alloy, and cryogenic measurements have only been made on a few.

This document describes a method for predicting the conductivity of aluminium alloys from limited information. A more detailed description is given in Reference [1].

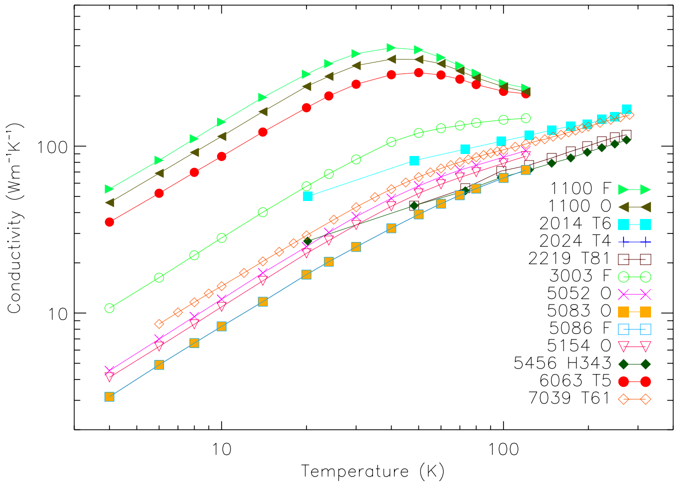
2 The Problem

Aluminium alloys are very useful for structural components, especially in aerospace applications:

- They have a high strength/density ratio
- They can be welded
- There are many different types available
    - There is thus lots of freedom to choose appropriate alloy for each application

For cryogenic applications, it is usually necessary to know the thermal conductivity

- This has been measured on only a few alloy types and tempers
- Results of an extensive literature search show measurements for:
    - 18 alloy types
    - 23 alloy/temper combinations (see Box 1)
    - Many are types no longer in use, or not commercial types or tempers
- Useful results, for commercially available types and tempers are shown below. There are:
    - 12 alloy types
    - 13 alloy/type combinations



Other problems:

- Most measurements are on US standard alloys
  - These are often not easily available in other countries
- Measurements were mostly made in the 60's
  - Composition and manufacturing processes for nominally identical alloy types are likely to have changed
  - Nominally equivalent alloys may have significantly different conductivities

3 Towards a solution

We can use the Wiedemann-Franz law:

- This gives thermal conductivity of a metal as a function of electrical resistivity
- There is good evidence that it works for aluminium alloys at low temperatures (to better than 10%; see Reference [1])
- So we just need to measure resisitivity

A commercial alloy is designated by the composition *and temper*.

Temper denotes the heat and/or mechanical treatment that has been applied

Examples: precipitation hardening, cold working

Alloys with the same composition but different temper can have very different conductivity – beware!

Chemical composition (6xxx=Si/Mg alloy, 061 denotes amounts of other elements)

Example: 6061 T6

Temper (T6=solution treated and aged)

BOX 1

- This is straightforward; it can be done with simple resistance bridge (e.g. one used for thermometry) and a dip probe in a helium dewar (a sample screwed to a piece of wood works just fine)

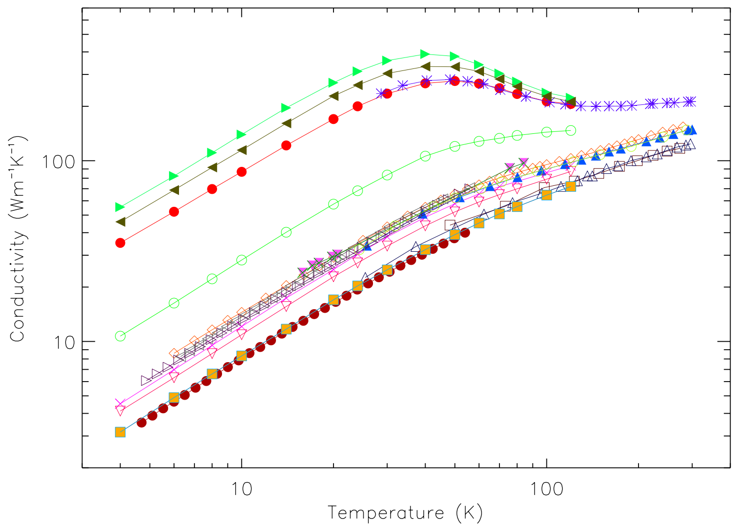
However:

- This is only useful at liquid helium temperatures (4 K) and below
  - At higher temperature, the relationship between thermal conductivity and electrical resistivity is not well known
- We need a method of extrapolating low temperature conductivity to higher temperatures

4 Conductivity fits

Looking at the literature data, there is a definite pattern

- The graph below shows all the available measurements which cover a useful temperature range, apart from three which do not appear to fit the pattern (assumed to be due to experimental error)



In general, the lines do not cross over.

- This suggests that any two alloys with the same conductivity at any one temperature should have the same conductivity at all temperatures, regardless of composition
  - Therefore we should be able to find an expression which fits all alloys with a single adjustable parameter

Such an expression has previously been produced for pure aluminium

- The equations are given in Reference [2]
  - These do *not* work for aluminium alloys (RRR<10)
  - But a modification extends them to work for alloys as well (see Box 2)
  - The modified equations produce fits to all the measurements shown above to an accuracy of 10% - adequate for most applications, and similar to likely experimental error for actual thermal conductivity measurements

The fit equations

Conductivity: $\kappa = \dfrac{1}{W_0 + W_i + W_{i0}}$

where:

$$W_i = P_1 T^{P_2}\left[1 + P_1 P_3 T^{(P_2+P_4)} \exp\left(-\left(\left(\frac{P_5}{T}\right)^{P_6}\right)\right)\right]^{-1} + W_c$$

$$W_0 = \frac{\beta}{T} \qquad\qquad W_{i0} = P_7 \frac{W_i W_0}{W_i + W_0}$$

$$W_c = -0.0005\ln\left(\frac{T}{330}\right)\exp\left[-\left(\frac{\ln\left(\frac{T}{380}\right)}{0.6}\right)^2\right] - 0.0013\ln\left(\frac{T}{110}\right)\exp\left[-\left(\frac{\ln\left(\frac{T}{94}\right)}{0.5}\right)^2\right]$$

T is temperature, and β is the adjustable parameter that changes with purity. The coefficients are given by:

$$P_1(\mathrm{RRR}^*) = \min\left(\alpha_1(\mathrm{RRR}^*)^{\beta_1}, P_{1\_pure}\right) \qquad \mathrm{RRR}^* \approx \frac{1}{\beta}$$

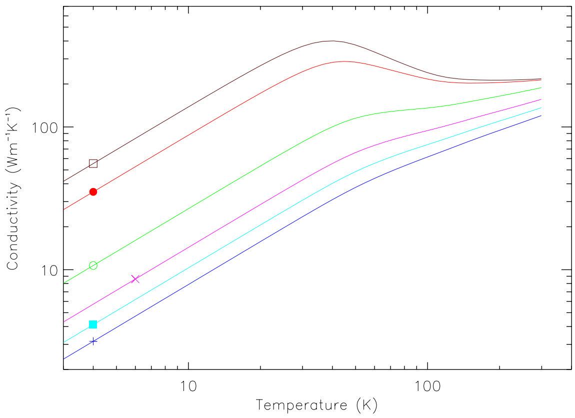
$$P_3(\mathrm{RRR}^*) = \max\left(\alpha_3(\mathrm{RRR}^*)^{\beta_3}, P_{3\_pure}\right)$$

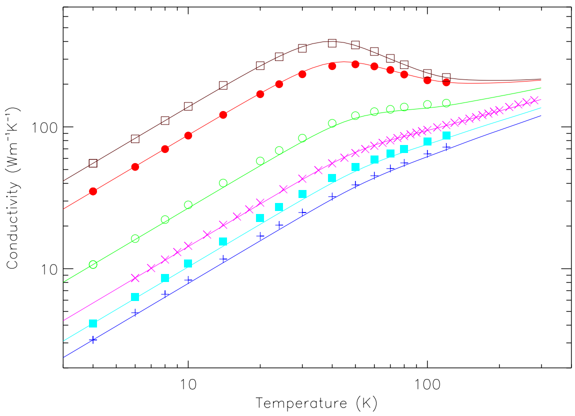
Parameter	Value
P_{1_pure}	4.716 x 10^{-8}
P_2	2.446
P_{3_pure}	623.6
P_4	-0.16
P_5	130.9
P_6	2.5
P_7	0.8168

Parameter	Value
α_1	2.958 x 10^{-8}
β_1	0.129
α_3	925.4
β_3	-0.167

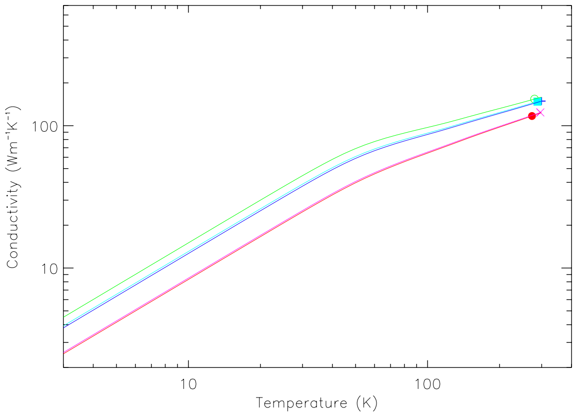
BOX 2

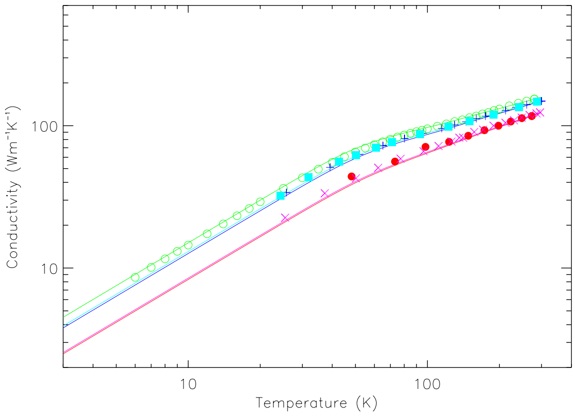
An example is shown below; the graph on the left shows a measured point along with the prediction; the graph on the right shows the entire datasets.





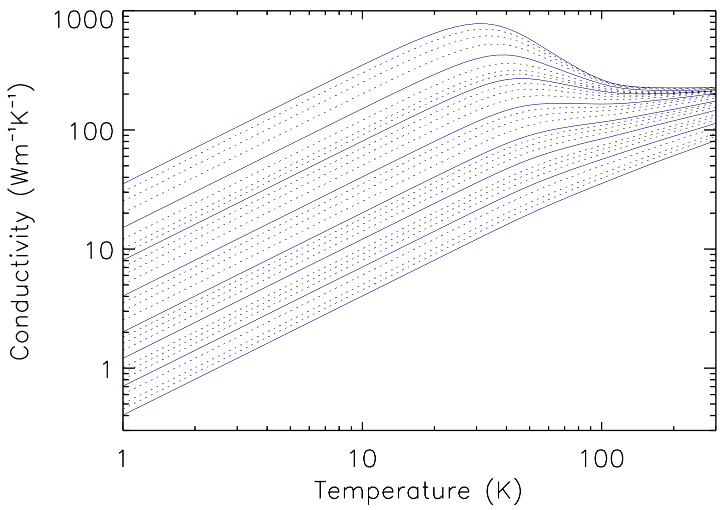
This works for high temperature points too, with a similar accuracy:





The graph below shows a family of conductivity curves generated using the equations in box 2.

- The *normal state* (see Box 3) conductivity of all aluminium alloys should fall on (or be interpolated between) these lines
- Each line corresponds to a different RRR value (see Box 4)



Aluminium alloys superconduct around 1 K

The transition temperature (below which they superconduct) varies with alloy composition and (probably) temper

Values seem to vary from <400 mK to at least 1.4 K with no obvious pattern other than that Al-Si and Al-Li alloys seem to have the highest value [1]

Therefore predicted conductivity values must be used with caution at temperatures below about 1.5 K

BOX 3

Metal conductivity is often described by RRR (residual resistivity ratio)

$$RRR = \frac{\text{Room temperature resistivity}}{\text{Low temperature resistivity}}$$

At sufficiently low temperatures, metal resistivity is usually constant – the "residual resistivity"

Since residual resistivity varies much more strongly with purity than room temperature resistivity, RRR is approximately inversely proportional to resistivity, and has more convenient values.

By the Wiedemann-Franz law, it is thus approximately proportional to conductivity.

BOX 4

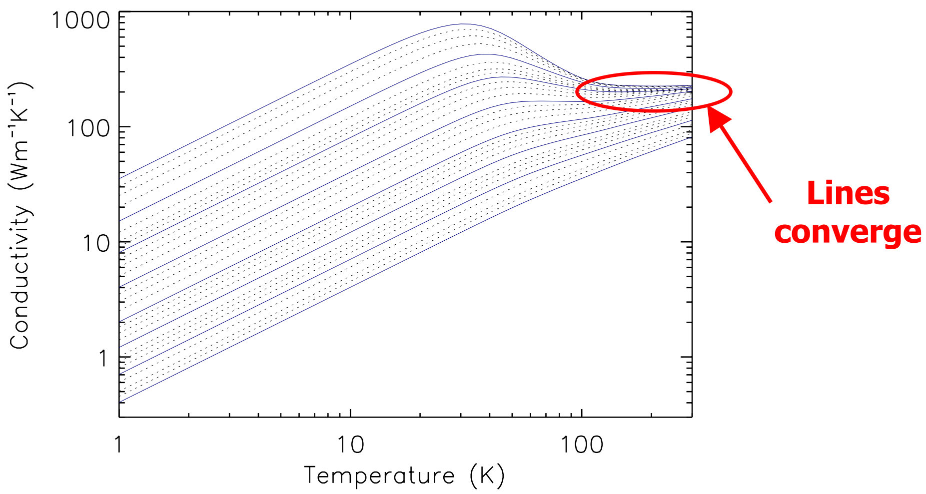
5 Uses

Uses of these equations include:

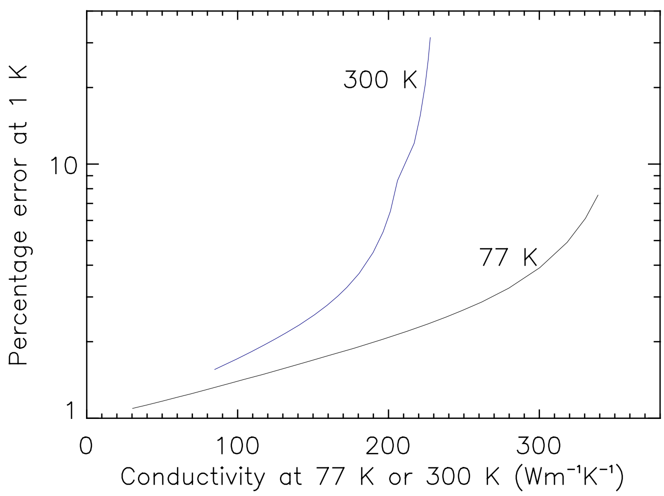
- Checking the likely validity of cryogenic measurements
- Extending cryogenic measurements to a larger temperature range
- Predicting the conductivity of unmeasured alloy types from a single low temperature resistivity measurement (easy and quick to make)
- Easily predicting the conductivity of actual materials used in an instrument to avoid errors due to sample variation, again from a simple resistivity measurement.

It is also possible to use room temperature datasheet conductivity values

- However, care must be taken with this
  - Lines converge at high temperature and purity (see below), so small errors at room temperature can lead to large errors at low temperature
  - Datasheet values may only be accurate to 10% or worse (errors are rarely quoted, and may be larger than expected)



  - The percentage error in the extrapolated conductivity at 1 K for a 1% error in measured conductivity at 77 K and 300 K is shown below:

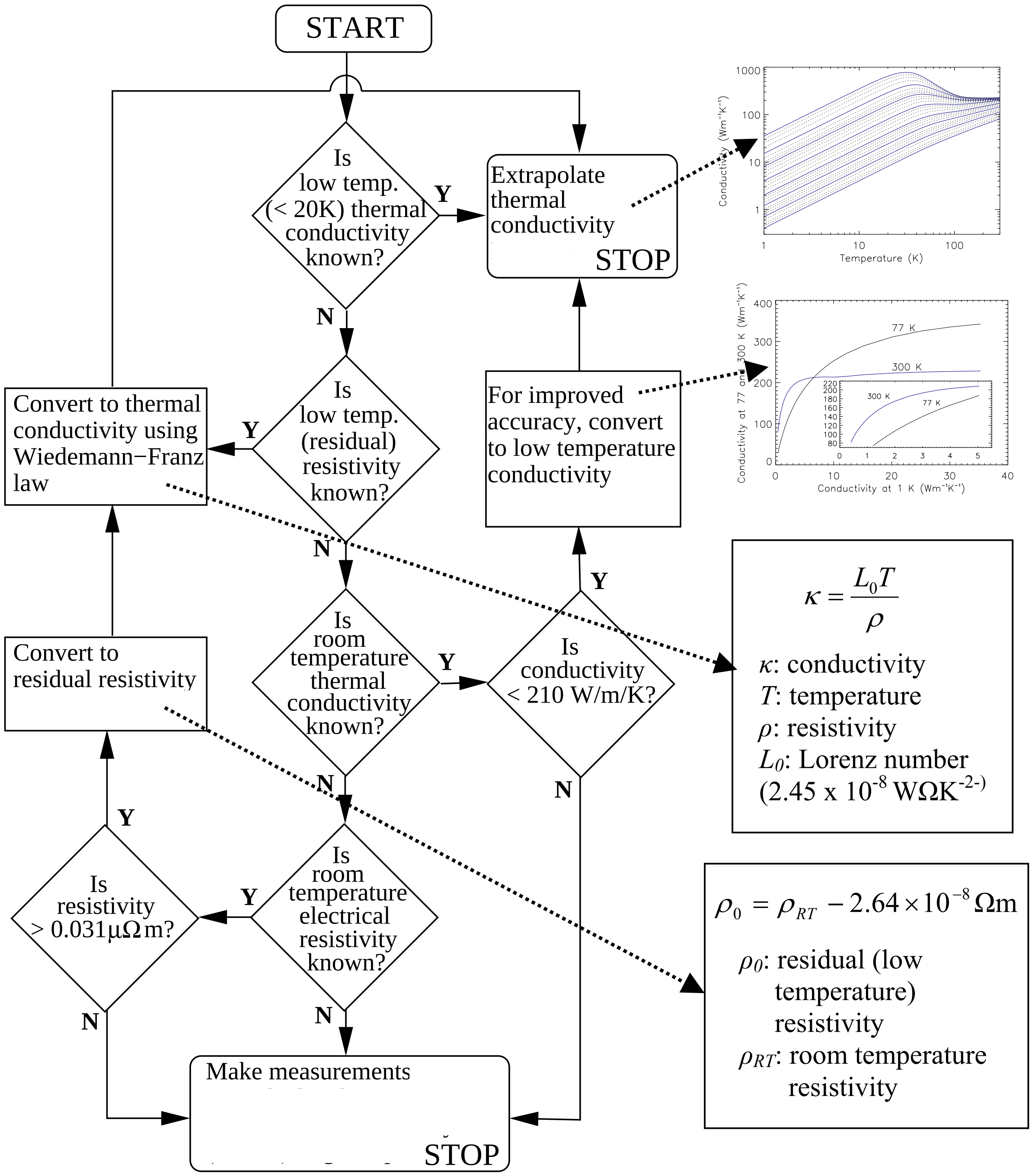


Room temperature resistivity values can also be used

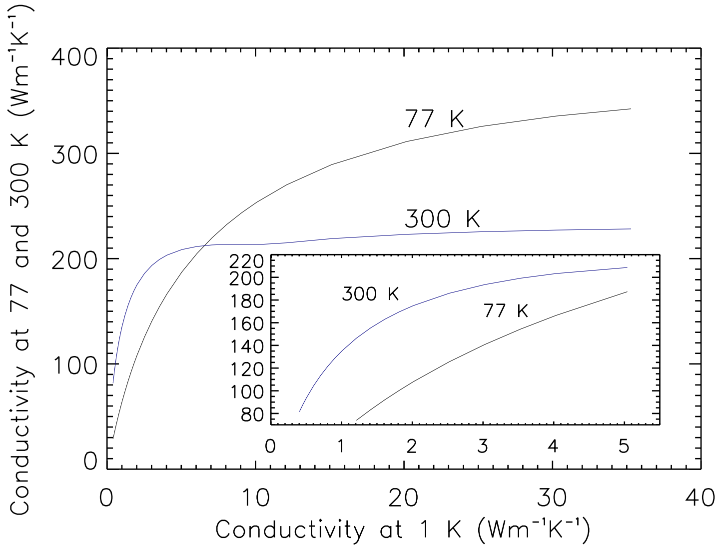
- The Wiedemann-Franz law can't be used at room temperature because it is not known well enough
- However, the residual resistivity can be determined for most aluminium alloys (room temperature resistivity > 0.031 μΩ m) by subtracting a constant value (Reference [1]).
- The Wiedemann-Franz law can then be used to determine the low temperature conductivity and thus conductivity at any temperature

6 Application

The flow chart below shows a suggested method for determining the conductivity of an aluminium alloy depending on what information is known



The graph below can be used to quickly obtain the low temperature conductivity if the value at 77 K or 300 K is known:



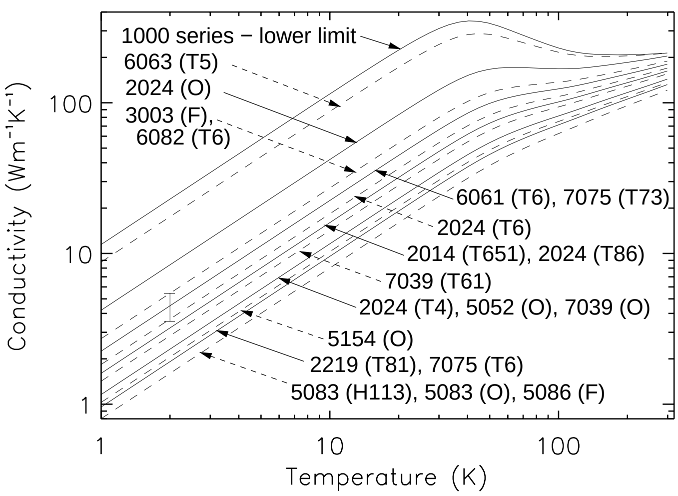
7 Conclusions

The equations presented here enable the thermal conductivity of any aluminium alloy to be extrapolated from ~1 K to 300 K based on any of:

- Thermal conductivity at a single temperature (including room temperature)
- Low temperature (residual) resitivity
- Room temperature resistivity

The graph below shows recommended values based on low temperature thermal and electrical measurements.

- More information is given in Reference [1].
- Many more could be added based on room temperature measurements.



A conductivity calculator using these results is available on the world-wide-web at http://reference.lowtemp.org/alkappa.html.

8 References

[1] A.L. Woodcraft, *"Predicting the thermal conductivity of aluminium alloys in the cryogenic to room temperature range"*, Cryogenics 45 (6) 421-431 (2005).

[2] Hust and Lankford, NBSIR 84-3007, National Bureau of Standards, Boulder Colorado (1984)