Is there a universal equation for fitting semiconductor thermistor resistivity?

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Abstract. Semiconductor bolometers and microcalorimeters operating at ultra-low temperatures are used in many applications, such as sub-millimetre and X-ray astronomy, dark matter searches, and double beta decay experiments. Various thermistor materials are used, primarily neutron transmutation doped germanium and ion implanted silicon. These materials both exhibit a similar variation of resistivity with temperature (R(T)), believed to be due to variable range hopping. A theoretically motivated equation provides good fits to the R(T) behaviour, with only two adjustable parameters. However, systematic deviations from these fits have been observed. It would be of practical use to obtain a single equation which could take such deviations into account without adding further adjustable parameters. An equation of this form has been proposed based on measurements on ion-implanted silicon [D. McCammon, in *Cryogenic Particle Detection*, Springer, Berlin, 2005]. Since this was also found to fit measurements on an NTD germanium thermistor, it was proposed as a general function for both materials. We show that it does not apply to NTD germanium in general, and propose an alternative two-parameter fitting function for this material. This enables greatly improved fits to be carried out compared with the standard function, without the penalty of introducing an extra parameter.

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INTRODUCTION

Neutron transmutation doped (NTD) [1] germanium thermistors are used in bolometers in various instruments currently operating, being commissioned (e.g. SPIRE on the Herschel Space Observatory [2] and HFI on the Planck Surveyor [3]) and under construction (e.g. CUORE [4]). Ion implanted silicon thermistors are also in use (e.g. PACS on Herschel). The relationship between current and voltage for both materials can be represented analytically to a high degree of accuracy [5], and the following expression (commonly known as the variable range hopping, or VRH, equation) is generally used:

$$R(T) = R_0 \exp\left(\frac{T_0}{T}\right)^p, \qquad (1)$$

where R is the resistance at temperature T, and T_0 and R_0 are constants which depend on the doping and, for R_0, on the thermistor dimensions. This equation has a theoretical motivation, and experimental results are generally in reasonably good agreement with a value of $p = 0.5$ [6]. Such a value of p also has a theoretical basis, but competing theories suggest different values; there is also expected to be another term in this equation which has a similar effect to changing the value of p [6].

Using an analytical temperature calibration function has two main advantages. Firstly, it eliminates the need for complex empirical functions such as Chebychev polynomials to fit calibration data. This reduces the number of calibration points required since accurate interpolation is possible between widely spaced points. Extrapolation outside the calibrated temperature range is even possible with some confidence. Secondly, it simplifies the modelling of devices such as bolometers, and enables analytical expressions to be written down for properties such as the responsivity [7].

An analytical expression is obviously only useful if it is the correct one. It has been recognized that deviations from equation (1) exist in both NTD germanium and ion-implanted silicon. We have found good agreement with equation (1) for NTD germanium so long as p is taken as a variable parameter [6, 8]. However, a different expression has been proposed, based on measurements on ion-implanted silicon [9]. This equation, referred to in Ref. [9] as the "Wouter function", has the form

$$\begin{aligned} R(T) &= R_0(T)\exp\left(\frac{T_0}{T}\right)^{\frac{1}{2}} + R_0'(T)\exp\left(\frac{T_0'}{T}\right)^{\frac{1}{2}} \qquad (2)\\ R_0' &= R_0\exp(2.522T_0^{-0.25} - 8.733),\\ T_0' &= 2.7148T_0 + 1.2328. \end{aligned}$$

This equation was found to fit measurements on silicon samples with a wide range of values of T_0, while

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equation (1) did not provide good fits, even if p was allowed to vary from 0.5 [9]. It was also found to provide a good fit to a sample of NTD germanium, and it was therefore proposed as a more general fitting function. It has the great advantage over equation (1) that it only contains two adjustable parameters.

ANALYSIS

First we examine the germanium thermistor described in Ref. [9], which we refer to here as the "McCammon" thermistor. Fig. 1 shows (as solid lines) the measured resistance as a function of temperature, plotted in the usual form so that equation (1) will produce a straight line. The top trace shows that a fit to equation (1) with $p = 0.5$ (plotted as •) produces a reasonable fit, but with obvious systematic errors. If instead the data is fitted only to temperatures above 200 mK and then extrapolated to lower temperatures (i.e. to the right of the dashed vertical line) (plotted as ○), a significant deviation is seen. The middle trace shows an extrapolation of equation (2) to temperatures below 200 mK (plotted as •), which is in very good agreement with the measurements. The bottom trace shows a fit to equation (1) over the full temperature range, allowing p to vary, which also produces a good fit, with a value of $p = 0.56$.

This can be seen more clearly in Fig. 2, where deviations between the measurement and the various fits are shown. As well as the fits described above, we also show the result of fitting equation (2) to the whole dataset,

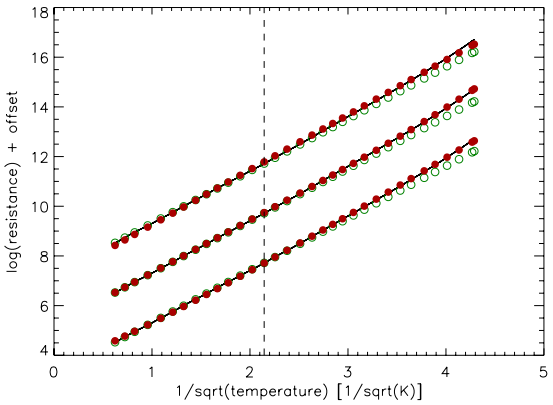

FIGURE 1. Measured data for the McCammon thermometer [9] (solid line), and a fit to equation (1) with $p = 0.5$ for temperatures above 200 mK, extrapolated to lower temperatures (○), plotted three times with an arbitrary vertical offset. Also shown, top grouping: fit to equation (1) with $p = 0.5$ over the full temperature range (•), middle grouping: fit to equation (2) to temperatures above 200 mK, extrapolated to lower temperatures (•), bottom grouping: fit to equation (1) over the full temperature range with p allowed to vary (•).

rather than just at high temperatures. It can be seen that this function fits well with both fitting methods, and describes the measurements extremely well, while equation (1) produces a fit which is somewhat worse, but generally accurate to ±1%.

Figure 2 shows similar results for thermometers 12-1 and F-1 from Ref [6], designed for use at 100 mK and 300 mK respectively. Again, equation (1) does not fit well with $p = 0.5$, but produces a much better fit if p is allowed to vary (in this case to 0.56 and 0.48 for thermometers 12-1 and F-1 respectively). However, equation (2) does not fit very well for these thermometers, either when fitted over the full temperature range or only to temperatures above 100 mK. Finally results are shown for a thermistor designed for use at much lower temperatures; thermometer NTD34B-1 from Ref. [8]. Again, equation (1) works well, in this case with $p = 0.63$, while equation (2) produces reasonable fits when fitted over the entire temperature range, but not when only fitted at high temperatures.

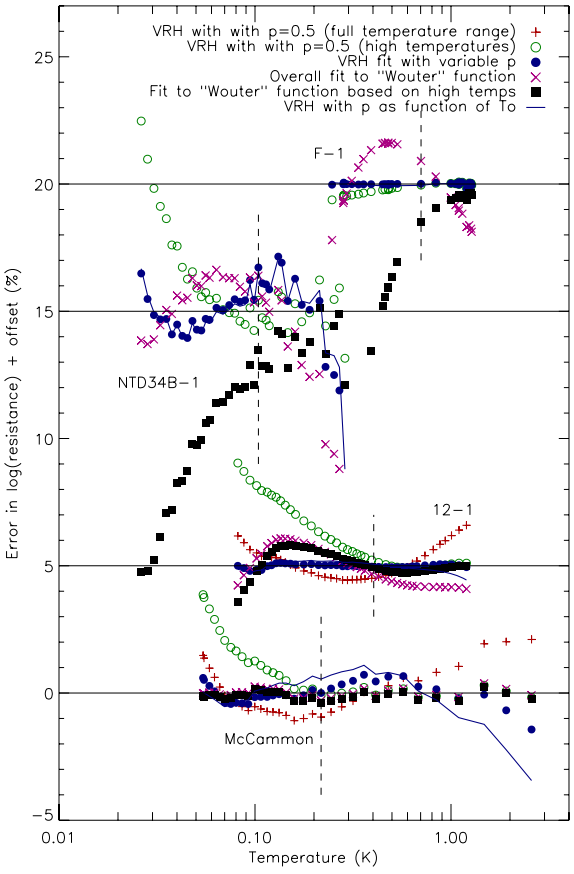

FIGURE 2. Deviations between the measurement and various fitting methods for several thermometers, as labelled. The vertical dashed lines show the temperature above which fits at "high temperatures" were carried out.

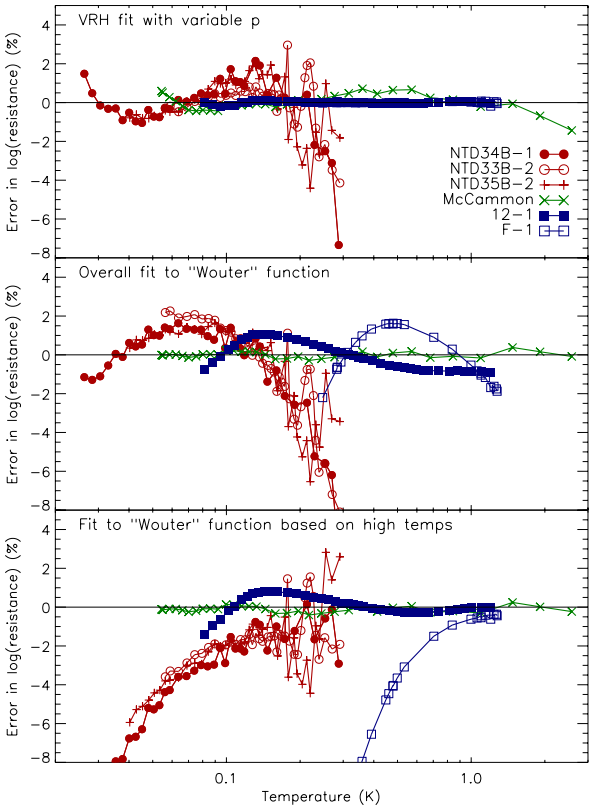

FIGURE 3. Results from Fig. 2, but separated into different graphs according to fit method, rather than thermometer. Thermometers NTD33B-2 and NTD35B-2 from Ref. [8] are also shown.

These results, along with those from two other thermistors from Ref. [8], are summarised in Fig. 3. The top panel shows fits over the full temperature range to equation (1), with p allowed to vary. The fits generally agree to at least $\pm 1\%$. The middle panel shows the result of fitting equation (2) over the full temperature range. Here the accuracy is somewhat less, with very evident systematic errors. The bottom panel shows the result of fitting equation (2) at high temperatures then extrapolating to the full temperature range as described above. This clearly produces very poor results in some cases. Equation (1) is therefore a better general function for NTD germanium, though equation (2) provides a somewhat better fit for the McCammon thermometer.

It therefore seems that we need to use different equations for silicon bolometers (equation (2)) to germanium bolometers (equation (1)). The disadvantage of equation (1) is that it has three adjustable parametes. We suggested a relationship between p and $T_0^{'}$ in Refs. [6] and [8], though the *bolometers* measured in Ref [6] did not agree so well. We chose to use $T_0^{'}$ (the value of T_0 obtained by fixing p to 0.5) since T_0 itself depends on p and is thus not an independent parameter. However, if instead we plot p against T_0, we see a better agreement for all thermometers measured, including bolometers. We can then attempt to carry out two-parameter fits with p taken as a function of T_0 rather than as a free parameter. The results from such fits, using the form

$$p = 0.625 - \frac{\log_{10}(T_0)}{12.9}, \qquad (3)$$

are shown in Fig. 2, where lines show fits from this two parameter fit, and points (plotted as $\times$) show fits from the full three parameter fit. While the results are not all quite as good as leaving p as a free parameter, they are generally accurate to a few percent. This would be acceptable in most situations, and is certainly a great improvement over the usual practice of fixing $p = 0.5$.

Figure 4 shows a comparison of two and three parameter fits for the thermistors already shown, along with thermistors in bolometers designed for operation at 300 mK [10]. We therefore show results for thermistors designed for operation at temperatures from 10 mK to above 300 mK, covering the range of operating temperatures at which such thermistors are normally used. The fits agree with the measurements to $\pm 2\%$ in almost every case. With the inclusion of the bolometers from Ref. [10] we are now showing results taken in four different systems, all in different laboratories and with entirely separate calibrations. We have not come across any germanium thermistors which cannot be fitted in this fashion, and therefore conclude that we have found a truly general two-parameter fit for NTD germanium.

Figure 5 shows the parameter p resulting from these fits as a function of $T_o^{'}$ taken from fits with $p = 0.5$. There is presumably some physical principle underlying this behaviour, but at present we can only take this as an empirical observation. Figure 5 shows that $p \simeq 0.5$ for thermistors designed for 300 mK operation; since the majority of instruments using NTD germanium operate at this temperature, it is perhaps not surprising that fits with $p = 0.5$ are used widely with so much success.

CONCLUSIONS

Deviations from the standard variable range hopping function (equation (1), with $p = 0.5$) have long been known in both silicon and germanium. In Ref. [9], an expression (the "Wouter function", equation (2)) with only two free parameters was derived which accounted for such deviations in ion-implanted silicon. Based on a measurement on NTD germanium, this was proposed as a general expression for materials exhibiting variable range hopping counduction. We have shown that this

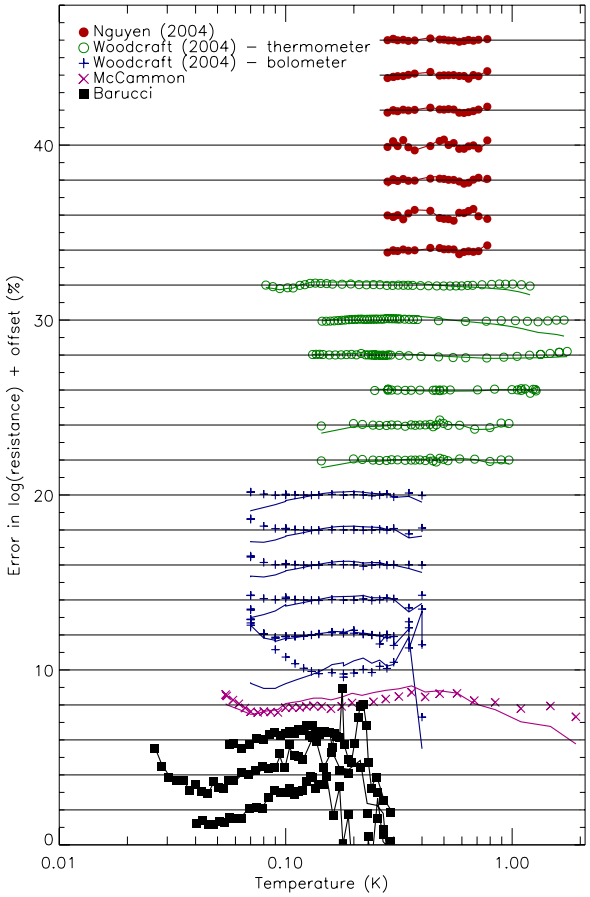

FIGURE 4. Results from thermometers shown earlier and others from Ref. [6] and [10]. As in Fig. 2, symbols show results from VRH fits (equation (1) with p as a free parameter), and lines show the equivalent two-parameter fit with p taken as a function of T_0. The horizontal lines correspond to a zero error for each measurement, and are spaced 2% apart.

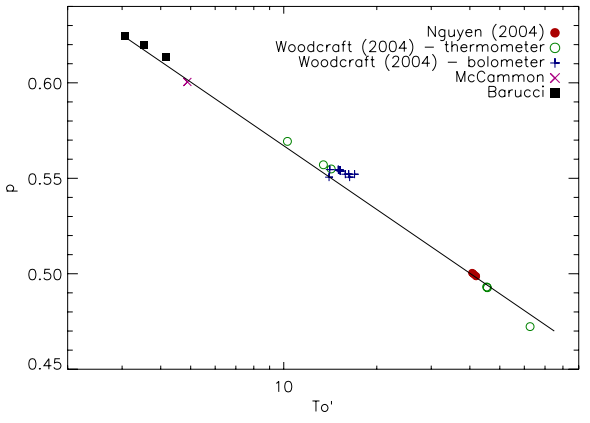

FIGURE 5. Values of p obtained from two-parameter VRH fits (with p a function of T_0) as a function of T_0' obtained from fits with $p = 0.5$, along with a fit (solid line).

expression does not provide good fits to NTD germanium in general, and thus is not truly universal.

However, we have shown that equation (1) does provide good fits to measurements on NTD germanium taken in three different laboratories and (with larger systematic errors) to the NTD germanium sample measured in Ref. [9], so long as the term p is taken as a free parameter rather than being fixed to $p = 0.5$. Such fits have the disadvantage of requiring three free parameters, unlike equation (2). However, we have found a relationship between p and T_0 in equation (1). By taking p as a function of T_0, we have obtained an expression with only two free parameters, but which successfully allows for variations from the standard two parameter fit with $p = 0.5$.

To answer the question posed in the title, we conclude that there is no known universal equation for fitting semiconductor thermistor resistivity in the variable range hopping regime. However, to complement the function already found for ion-implanted silicon, we have found a two-parameter fitting function which applies to NTD germanium thermistors designed for use over the full range of temperatures at which they are normally employed.

Since the thermistors in a given instrument normally have near identical values of T_o, we suggest that a suitable value of p is chosen using equation (3), and fits are then carried out using equation (1). This is as simple to apply as the customary approach of taking $p = 0.5$, and is likely to be considerably more accurate.

REFERENCES

1. E. E. Haller, *Infrared Phys.* **25**, 257–266 (1985).
2. G. L. Pilbratt, *Proc. SPIE* **7010**, 701002 (2008).
3. N. Mandolesi, and M. Villa, "FIRST/Planck mission," in *ICMT/99. Proceedings of the 16th IEEE Instrumentation and measurement technology conference, IEEE, Piscataway, NJ, USA*, edited by V. Piuri, and M. Savino, 1999, vol. 2, pp. 975–80.
4. P. Gorla et al., *Nucl. Phys. B (Proc. Suppl.)* **150**, 214–218 (2006).
5. P. L. Richards, *J. Appl. Phys.* **76**, 1–24 (1994).
6. A. L. Woodcraft, R. V. Sudiwala, E. Wakui, and C. Paine, *J. Low Temp. Phys.* **134**, 925–944 (2004).
7. R. V. Sudiwala, M. J. Griffin, and A. L. Woodcraft, *Int. J. Inf. Mill. Waves* **23**, 545–573 (2002).
8. M. Barucci, J. Beeman, E. Olivieri, E. Pasca, L. Risegari, and G. Ventura, *Physica B* **368**, 139–142 (2005).
9. D. McCammon, "Semiconductor Thermistors" in *Cryogenic Particle Detection*, Springer, Berlin, 2005, pp. 35-61.
10. H. T. Nguyen, J. J. Bock, P. Ringold, J. Battle, S. C. Elliott, A. D. Turner, M. Weilert, V. V. Hristov, B. Schulz, K. Ganga, L. Zhang, J. W. Beeman, P. A. R. Ade, and P. C. Hargrave, *Proc. SPIE* **5498**, 196–207 (2004).